Nonequilibrium Field Theories and Stochastic Dynamics

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1 Intro: Fokker-Planck Equation

Fokker-Planck Equation, which can describe the evolution of the probability density function in a continuous Markov process, provides a universal and powerful mathmatical framework.

The general Fokker-Planck Equation in the one dimensional system can be written as:

$$\frac{\partial P(x,t)}{\partial t} = -\frac{\partial}{\partial x}[A(x)P(x,t)] + D(x)\frac{\partial^2}{\partial x^2}P(x,t), \tag{1}$$

Where $P(x,t)$ is the probabilty density function of the par tical on the position x and at the moment t. The equation has two core, which describe the two fundamental physical process to drive the partical moving, respectively.

- **Drift Term:** Described by the coefficient $\mathbf{A}(\mathbf{x})$. This term represents the deterministic, systematic influence acting on the particle, and can be understood as a force. It describes the average velocity of the particle at position $\mathbf{x}$, tending to "push" the center of the probability distribution towards a specific direction or location. For example, gravity causes a particle to drift downwards, and the restoring force of a spring causes a particle to drift toward the equilibrium position.
- **(Diffusion Term:** Described by the coefficient $\mathbf{D}(\mathbf{x})$. This term represents the random, incidental influence, quantifying the intensity of the random "kicks" experienced by the particle. It originates from the thermal fluctuations of the system's environment (such as molecules in a liquid) and causes the probability distribution to "spread out" or diffuse over time. The diffusion coefficient $\mathbf{D}(\mathbf{x})$ is directly related to the variance of the particle's displacement.

So, where does this equation come from? It is not derived out of thin air but is based on profound physical assumptions. The Fokker-Planck equation can be derived from the more fundamental Chapman-Kolmogorov equation via the Kramers-Moyal expansion. The crucial step in this derivation is making a physical simplification: we assume that within an infinitesimal time step Δt, only the first moment (the mean displacement, related to drift) and the second moment (the mean-square displacement, related to diffusion) of the particle's displacement are significant, while all higher-order moments (such as Δx^3, etc.) can be neglected.This assumption is extremely reasonable physically, as it corresponds precisely to a class of processes where the system's evolution is the accumulation of a large number of tiny, independent random events (for example, the countless collisions between a Brownian particle and water molecules). In such a case, the Central Limit Theorem guarantees that the total displacement tends toward a Gaussian process. Therefore, by starting with the Fokker-Planck equation, we are effectively limiting our scope of study to a class of physical phenomena driven by smooth deterministic forces and fine-grained, Gaussian-like noise

2 Wiener Process—The Prototype of Pure Diffusion

The Wiener Process is the rigorous mathematical formulation of Brownian Motion, serving as a cornerstone of stochastic process theory. This process describes the random movement of a particle driven solely by thermal fluctuations from the environment, without the guidance of any external forces or potential fields.

2.1 The basical property

We can precisely characterize the Wiener process by defining the moments of the particle's displacement **dx** over an infinitesimal time interval **dt**

$$A = \lim_{\Delta t \to 0} \frac{\{\Delta x\}}{\Delta t} = 0. \tag{2}$$

The physical meaning of this condition is that the particle's movement has no systematic directional preference. At any given moment, the probability of it moving left is exactly equal to the probability of it moving right. Consequently, when averaged over a large number of particles, their mean displacement is always zero. Although the particle is continuously moving, "on average, it goes nowhere.

$$B = \lim_{\Delta t \to 0} \frac{\{(\Delta x)^2\}}{\Delta t} = 2D. \tag{3}$$

where **D** is a positive constant, known as the diffusion coefficient. This condition is the core characteristic of the Wiener process. It shows that the particle's mean squared displacement (MSD) is proportional to time: $\langle \mathbf{dx}^2 \rangle \propto \mathbf{dt}$. This reveals a profound essence of the diffusion process: although the particle's average position remains unchanged, the extent of its movement (or the uncertainty in its position) grows steadily with the square root of time.The dimension of the diffusion coefficient **D** is [Length]^2/[Time], and it quantifies the speed at which this region of uncertainty expands. Substituting these two conditions ($\langle \mathbf{dx} \rangle = 0$ and **D** is a constant) into the general Fokker-Planck equation, we obtain the partial differential equation that describes the Wiener process, known as the Diffusion Equation:

$$\frac{\partial P(x,t|x_0,t_0)}{\partial t} = D \frac{\partial^2}{\partial x^2} P(x,t|x_0,t_0) \tag{4}$$

2.2 Diffusion equation and its solution

The Diffusion Equation describes how a probability distribution evolves over time purely due to random motion in the absence of drift. For a particle that is definitely located at $\mathbf{x}_0$ at time $t = 0$ (i.e., the initial condition is $P(\mathbf{x}, 0) = \delta(\mathbf{x} - \mathbf{x}_0)$), the solution to this equation is a Gaussian function (Normal Distribution):

$$P(\mathbf{x}, t) = \frac{1}{\sqrt{4\pi \mathbf{D} t}} \exp\left(-\frac{(\mathbf{x} - \mathbf{x}_0)^2}{4\mathbf{D}t} \right) \tag{5}$$

This solution perfectly embodies the statistical properties of the Wiener process:

- **Mean:** The center (peak) of this Gaussian distribution consistently remains at the initial position $\mathbf{x}_0$. This is consistent with the property of zero drift, i.e., $\langle \mathbf{dx} \rangle = 0$.
- **Variance:** The variance of this Gaussian distribution is $\sigma^2 = 2\mathbf{D}t$. The variance grows linearly with time, meaning the width of the probability distribution (measured by the standard deviation σ) grows with the square root of time. This is the direct manifestation of the mean-squared displacement growing linearly with time. As time progresses, the area where the particle can be found becomes wider, and our prediction of its location becomes increasingly uncertain

2.3 Simulating the brown motion using python

Computer simulations allow us to more intuitively grasp the random trajectory of a single particle and the collective behavior of an ensemble composed of many particles. To simulate the Wiener Process, we typically use its corresponding Stochastic Differential Equation (SDE):

$$\mathbf{dx} = \sqrt{2\mathbf{D}} \mathbf{dW}(t) \tag{6}$$

Here, **x** is the particle's position, and $\mathbf{dW}(t)$ is the infinitesimal increment of the Wiener Process. This increment is a Gaussian random variable with a mean of 0 and a variance of **dt**.We can numerically solve this SDE using the Euler-Maruyama method. This is a simple and effective method for discretizing a continuous-time process. Its update rule is as follows:

$$\mathbf{x}_{i+1} = \mathbf{x}_i + \sqrt{2\mathbf{D}\Delta t}\mathbf{Z} \tag{7}$$

Where $\mathbf{Z}$ is a random number drawn from a standard normal distribution with a mean of 0 and a variance of 1. This update rule directly implements the core properties of the Brownian motion increment: the mean of the displacement is 0, and the variance is $2\mathbf{D}\Delta t$.

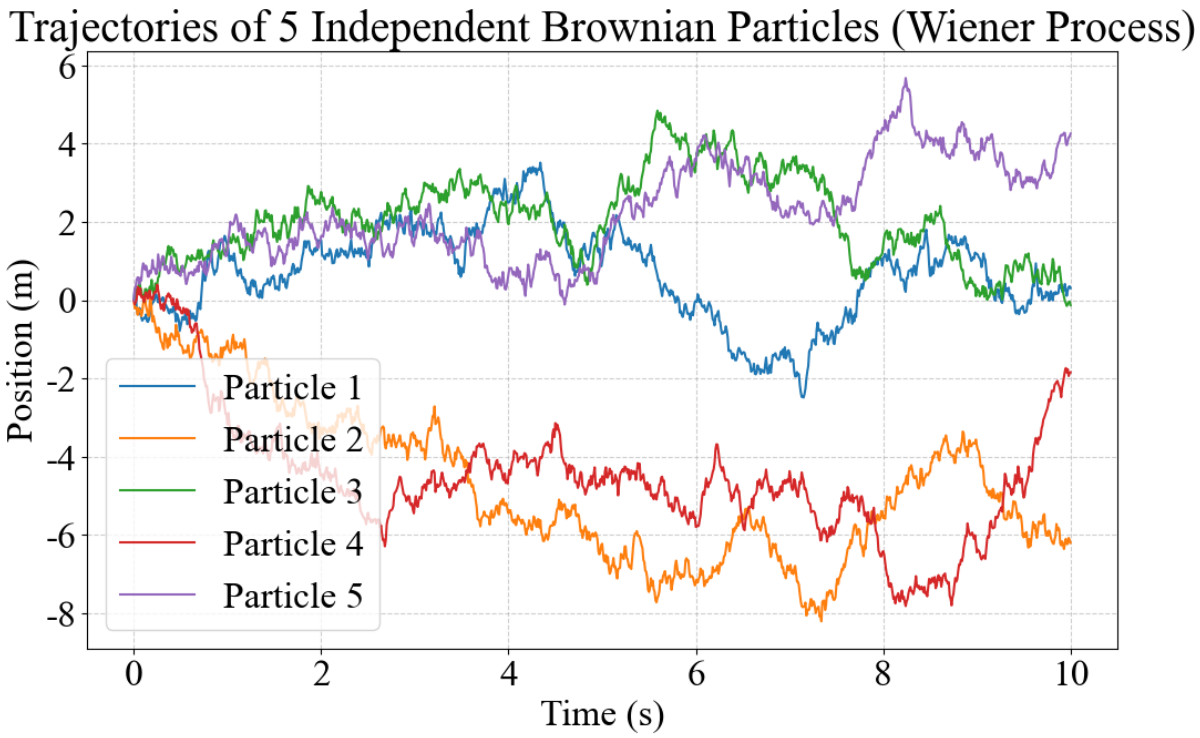

Figure 1:

As can be seen from the figure, the trajectory of every single particle is highly irregular and "jagged." This visually demonstrates a profound mathematical property of the Wiener process: its paths are continuous everywhere but nowhere differentiable. Continuity means that the particle does not jump instantaneously, while non-differentiability means that an instantaneous velocity cannot be defined at any point in time.If we zoom in on any small segment of the trajectory, it still looks as convoluted as the whole, never becoming smoother. The reason behind this is that the displacement $\mathbf{dx}$ is proportional to $\sqrt{\mathbf{dt}}$, and therefore the estimated instantaneous velocity, $\mathbf{dx/dt}$, diverges as $\mathbf{dt} \to 0$. This also explains why we write the SDE using the differential form $\mathbf{dx}$ rather than the derivative form $\mathbf{dx/dt}$

While individual trajectories are random, the collective behavior of a large number of particles is deterministic and follows the predictions of the Diffusion Equation. The following code simulates an ensemble consisting of 10,000 particles and compares their position distribution at different times with the theoretical Gaussian solution

This simulation demonstrates the unity of theory and practice. Although the path of every single particle is unpredictable, the probability distribution of the entire particle ensemble evolves in a completely deterministic way: starting from a sharp spike concentrated at the initial position (a Dirac delta function), it gradually broadens into an increasingly wide Gaussian distribution.

The histogram obtained from the simulation shows a high degree of agreement with the analytical solution of the Diffusion Equation (the Gaussian distribution). This powerfully demonstrates how the macroscopic deterministic law of the Fokker-Planck equation emerges from the microscopic random walk.

3 Ornstein-Uhlenbeck process – constrained diffusion

The Wiener process describes completely free diffusion, but in many physical and biological systems, random motion is subject to some form of constraint. The Ornstein-Uhlenbeck (O-U) process is the

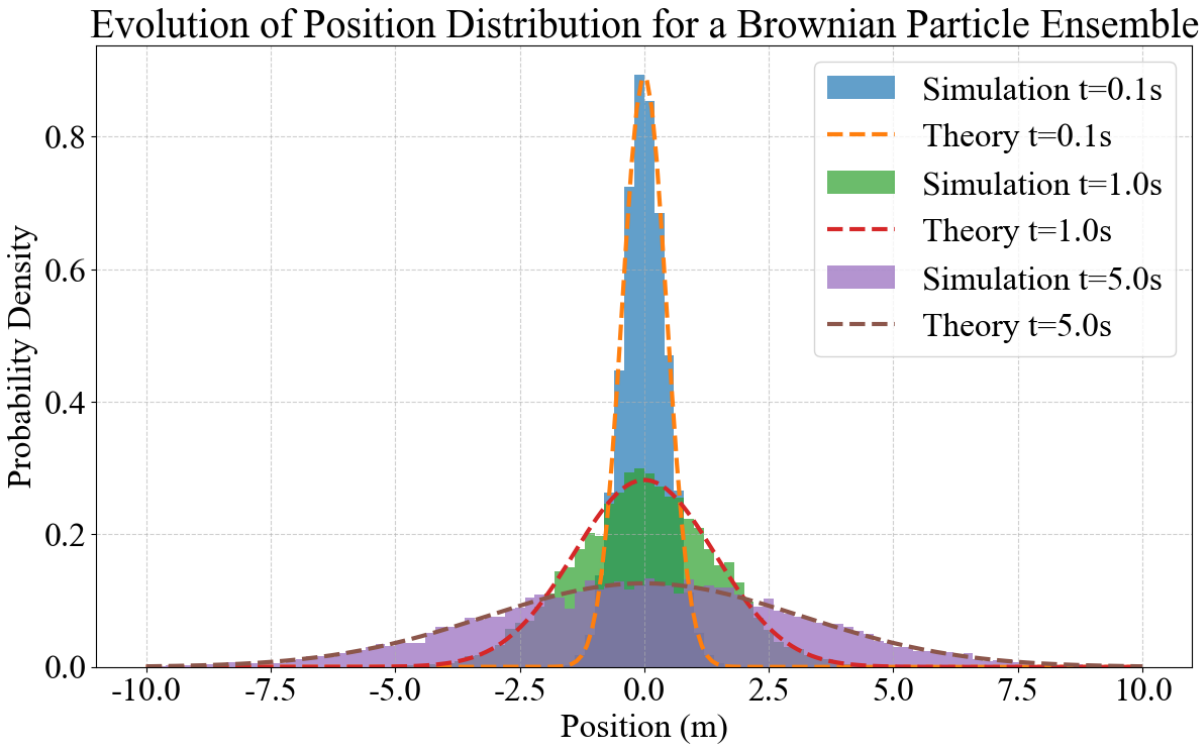

Figure 2:

most classic model describing such systems. It introduces a restoring force that pulls particles toward their equilibrium position, thus counteracting the diffusion effect.

3.1 Brownian particles in a harmonic oscillator potential well

Imagine a tiny particle suspended in a liquid, subjected not only to random collisions with water molecules (Brownian motion) but also bound to its origin by an invisible "spring." This binding can be described by a harmonic oscillator potential (harmonic potential):

$$U(x) = \frac{1}{2}kx^2 \tag{8}$$

Where k is the spring constant, representing the strength of the potential well. Now, let's analyze the two forces acting on this particle:

Deterministic drift (restoring force of the spring): The potential energy $U(\mathbf{x})$ generates a linear restoring force $F_{\text{restore}} = -\nabla U = -k\mathbf{x}$. This force always points toward the minimum potential energy ($\mathbf{x} = 0$) and is proportional to the distance from the center. In a fluid environment with a low Reynolds number (i.e., slow and small-scale motion), the particle's inertia can be neglected. This restoring force is then balanced by the fluid's Stokes' drag. Stokes' Law tells us that the drag force $\mathbf{F}_{\text{drag}}$ exerted on a spherical particle of radius R moving at velocity $\dot{\mathbf{x}}$ in a fluid with viscosity η is $F_{\text{drag}} = \zeta\dot{\mathbf{x}}$, where $\zeta = 6\pi\eta R$ is the **friction coefficient**.

The force balance equation is $F_{\text{restore}} + F_{\text{drag}} = 0$, i.e.:

$$-k\mathbf{x} = \zeta\dot{\mathbf{x}}$$

Upon rearrangement, we obtain the deterministic equation of motion for the particle in the absence of random noise:

$$\dot{\mathbf{x}} = -\frac{k}{\zeta}\mathbf{x} = -\gamma\mathbf{x}$$

Here, $\gamma = k/\zeta$ is called the **relaxation rate**, and its reciprocal, $\tau = 1/\gamma$, is the system's characteristic relaxation time. This equation describes how the particle exponentially returns to its equilibrium position. Therefore, the drift term of the Ornstein-Uhlenbeck (O-U) process is a function linearly dependent on position: $A(\mathbf{x}) = -\gamma\mathbf{x}$ (4). This drift term exhibits the characteristic of **mean-reversion**.

Random diffusion (thermal fluctuations of fluids): Similar to the Wiener process, random collisions from surrounding fluid molecules still occur. We assume that the strength of this random force is independent of the particle's position, therefore the diffusion term remains a constant: $B = 2D$

3.2 Governing equations and their solutions

Substituting the derived drift term $A(\mathbf{x}) = -\gamma\mathbf{x}$ and the diffusion term D into the general Fokker-Planck equation yields the controlling equation for the O-U process (4):

$$\frac{\partial P(\mathbf{x},t)}{\partial t} = \frac{\partial}{\partial \mathbf{x}}\left[\gamma\mathbf{x}P(\mathbf{x},t)\right] + D\frac{\partial^2 P(\mathbf{x},t)}{\partial \mathbf{x}^2}$$

This equation vividly describes a "tug-of-war": The first term (the drift term) attempts to compress the probability distribution towards the origin, making it narrower and taller; the second term (the diffusion term), conversely, attempts to spread the probability distribution outwards, making it wider and flatter. The final state of the system will be the result of these two effects reaching a dynamic equilibrium.

To solve this partial differential equation, the lecture uses a very powerful technique—the **method of characteristics**—combined with the Fourier transform. We will now detail the solution process:

We define the Fourier transform of the probability density $P(\mathbf{x},t)$, which is its **characteristic function** $\phi(\mathbf{k},t)$:

$$\phi(\mathbf{k},t) := \int_{-\infty}^{\infty} d\mathbf{x}\, e^{i\mathbf{k}\mathbf{x}} P(\mathbf{x},t) \tag{9}$$

The advantage of the Fourier transform is that it converts the spatial derivative $\partial/\partial\mathbf{x}$ into multiplication by the wave number $\mathbf{k}$ (specifically, multiplication by $i\mathbf{k}$). Applying this transform makes the original partial differential equation much simpler.

By applying the Fourier transform to both sides of the O-U FPE (Fokker-Planck Equation) and utilizing integration by parts and the transformation properties, we obtain the equation satisfied by $\phi(\mathbf{k},t)$:

$$\frac{\partial\phi(\mathbf{k},t)}{\partial t} + \gamma\mathbf{k}\frac{\partial\phi(\mathbf{k},t)}{\partial\mathbf{k}} = -D\mathbf{k}^2\phi(\mathbf{k},t) \tag{10}$$

This is a first-order linear partial differential equation concerning $\phi(\mathbf{k},t)$. We look for special curves in the $(\mathbf{k},t)$ space, called **characteristics**, along which the above partial differential equation can be simplified into an ordinary differential equation (ODE). These characteristic curves are given by the equation $d\mathbf{t}/1 = d\mathbf{k}/(\gamma\mathbf{k})$, which, upon integration, yields the characteristic curve equation:

$$\frac{d\mathbf{k}}{d\mathbf{t}} = -\gamma\mathbf{k} \implies \mathbf{k}(t) = \mathbf{k}_0 e^{-\gamma t} \tag{11}$$

Where $\mathbf{k}_0$ is the initial wave number at $t = 0$. Along such a characteristic curve, the total derivative of $\phi(\mathbf{k}(t),t)$ with respect to time is:

$$\frac{d\phi}{d\mathbf{t}} = \frac{\partial\phi}{\partial\mathbf{t}} + \frac{\partial\phi}{\partial\mathbf{k}}\frac{d\mathbf{k}}{d\mathbf{t}} = \frac{\partial\phi}{\partial\mathbf{t}} + \frac{\partial\phi}{\partial\mathbf{k}}(-\gamma\mathbf{k}) \tag{12}$$

By comparing this with the transformed PDE, we find that along the characteristic lines, the equation simplifies to:

$$\frac{d\phi}{d\mathbf{t}} = -D\mathbf{k}(t)^2\phi(t) \tag{13}$$

This is a simple first-order ordinary differential equation, and its solution is:

$$\phi(t) = \phi(0)\exp\left(-D\int_0^t d\mathbf{t}'\mathbf{k}(\mathbf{t}')^2\right) \tag{14}$$

We substitute $\mathbf{k}(t') = \mathbf{k}_0 e^{-\gamma t'}$ into the integral and assume that at the initial time $t = 0$ the particle is located at $\mathbf{x}_0$. In this case, $P(\mathbf{x}, 0) = \delta(\mathbf{x} - \mathbf{x}_0)$, and the corresponding initial characteristic function is $\phi(\mathbf{k}, 0) = e^{i\mathbf{k}\mathbf{x}_0}$. Hence $\phi(0) = e^{i\mathbf{k}_0\mathbf{x}_0}$.

The integral evaluates as:

$$\int_0^t d\mathbf{t}' \mathbf{k}_0^2 e^{-2\gamma t'} = \mathbf{k}_0^2 \left[\frac{e^{-2\gamma t'}}{-2\gamma} \right]_0^t = \frac{\mathbf{k}_0^2}{2\gamma}(1 - e^{-2\gamma t}) \tag{15}$$

Thus, the solution for $\phi(t)$ is:

$$\phi(t) = \exp\left(i\mathbf{k}_0\mathbf{x}_0 - \frac{D\mathbf{k}_0^2}{2\gamma}(1 - e^{-2\gamma t}) \right) \tag{16}$$

Finally, we express $\mathbf{k}_0$ in terms of $\mathbf{k}(t)$, which is $\mathbf{k}_0 = \mathbf{k}(t)e^{\gamma t}$, and substitute this back into the above equation to obtain the final solution for the characteristic function:

$$\phi(\mathbf{k}, t) = \exp\left(i\mathbf{k}\mathbf{x}_0 e^{-\gamma t} - \frac{D\mathbf{k}^2}{2\gamma}(1 - e^{-2\gamma t}) \right) \tag{17}$$

The characteristic function $\phi(\mathbf{k}, t)$ has a very specific form: it is the Fourier transform of a Gaussian function. The standard form for the characteristic function of a general Gaussian distribution is:

$$\phi_{\text{Gaussian}}(\mathbf{k}) = \exp\left(i\mathbf{k}\langle\mathbf{x}\rangle - \frac{1}{2}\mathbf{k}^2 \text{Var}(\mathbf{x}) \right) \tag{18}$$

By comparing our solution with this standard form term by term, we can directly read off the evolution rules for the particle's mean position and variance over time:

Mean:

$$\langle\mathbf{x}(t)\rangle = \mathbf{x}_0 e^{-\gamma t} \tag{19}$$

The average position of the particle ensemble starts at the initial position $\mathbf{x}_0$ and exponentially decays towards the center of the potential at $\mathbf{x} = 0$. This is exactly consistent with the deterministic equation of motion when noise is completely ignored. This shows that the random noise, while making every single particle trajectory jagged, does not change the overall average trend of the system.

Variance:

$$\text{Var}[\mathbf{x}(t)] = \frac{D}{\gamma}(1 - e^{-2\gamma t}) \tag{20}$$

The evolution of the particle position's variance (i.e., the width of the distribution) is more interesting.

- **In the initial stage ($t \to 0$):** Using the Taylor expansion $e^{-2\gamma t} \approx 1 - 2\gamma t$, the variance approximates to $\text{Var}[\mathbf{x}(t)] \approx (D/\gamma)(2\gamma t) = 2Dt$. In the short time scale, the particle has not yet felt the restoring force, and its behavior is exactly the same as free Brownian motion (Wiener process).
- **As time goes by ($t \to \infty$):** The exponential term $e^{-2\gamma t}$ approaches 0, and the variance converges to a constant steady-state value:

$$\text{Var}_{\text{ss}} = \lim_{t\to\infty} \text{Var}[\mathbf{x}(t)] = \frac{D}{\gamma}$$

This **steady state** is the most critical characteristic of the O-U process. It means that the system will eventually reach a dynamic equilibrium: the tendency of the restoring force to pull particles toward the center is perfectly canceled out by the tendency of diffusion to spread the particles outward. The probability distribution no longer changes over time, forming a stable Gaussian distribution centered at the origin with a variance of D/γ. The system "forgets" its initial state $\mathbf{x}_0$.

3.3 Simulating bound particles

Similar to the Wiener process, we can gain an intuitive understanding by numerically simulating the SDE corresponding to the O-U process:

$$d\mathbf{X}(t) = -\gamma \mathbf{X}(t) d\mathbf{t} + \sqrt{2D} d\mathbf{W}(t) \tag{21}$$

Its Euler-Maruyama discrete form is:

$$\mathbf{X}(t + \Delta t) = \mathbf{X}(t) - \gamma \mathbf{X}(t) \Delta t + \sqrt{2D\Delta t} \cdot \mathcal{N}(0, 1) \tag{22}$$

This update rule incorporates both the deterministic drift term (which moves the particle towards the origin) and the stochastic diffusion term.

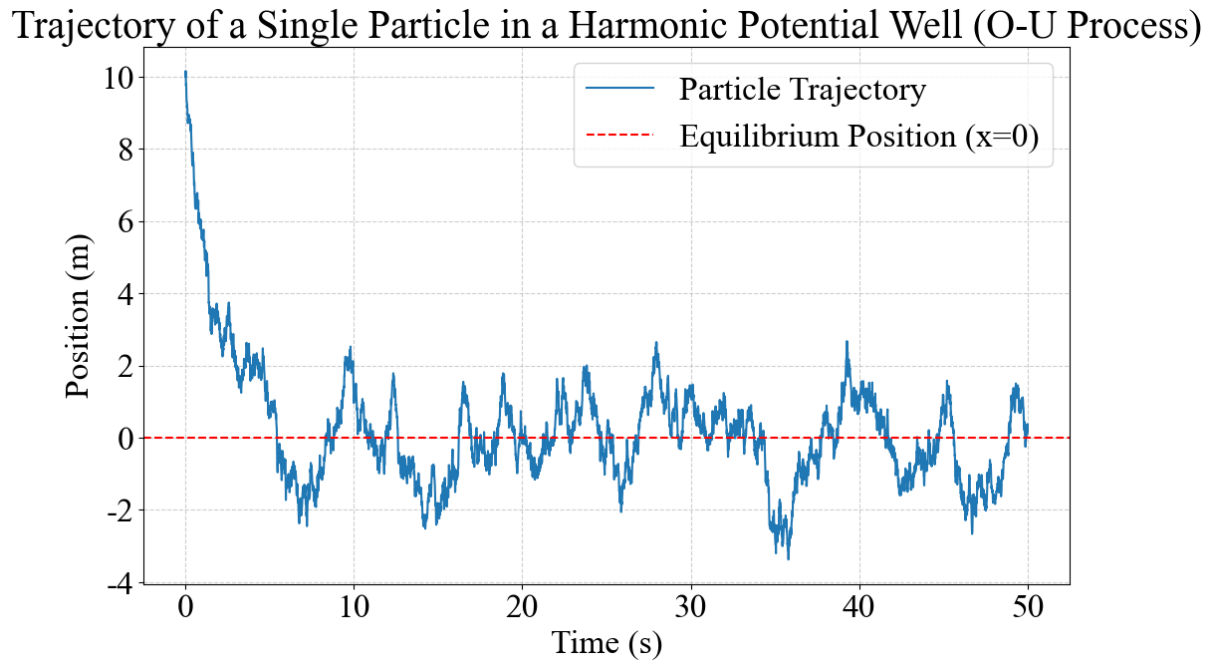

Figure 3:

As can be clearly seen from the trajectory diagram, the particle oscillates randomly around the origin. When it accidentally drifts to a point far from the origin, the strong restoring force (drift term) more significantly "pulls" it back. Ultimately, the particle's motion is confined to a finite region centered on the origin.

We now simulate a particle ensemble and verify whether its mean and variance are completely consistent with our analytical solution.

The calculated ensemble mean and variance coincide with the curves of the theoretical formulas. We can see how the mean decays exponentially to 0, and how the variance starts to grow from 0 and eventually saturates to a steady-state value.

4 The Einstein-Stokes relation: A profound connection between fluctuations and dissipation.

We already know that when time tends to infinity ($t \to \infty$), the Brownian particle system in the harmonic oscillator potential will reach a **steady state**. In this state, the system completely "forgets" its initial position $\mathbf{x}_0$, and its probability distribution is a time-independent Gaussian distribution centered at the origin. The variance of this steady state, which is also the mean squared value of the particle's position, is:

$$\langle \mathbf{x}^2 \rangle_{\text{ss}} = \text{Var}_{\text{ss}} = \frac{D}{\gamma} \tag{23}$$

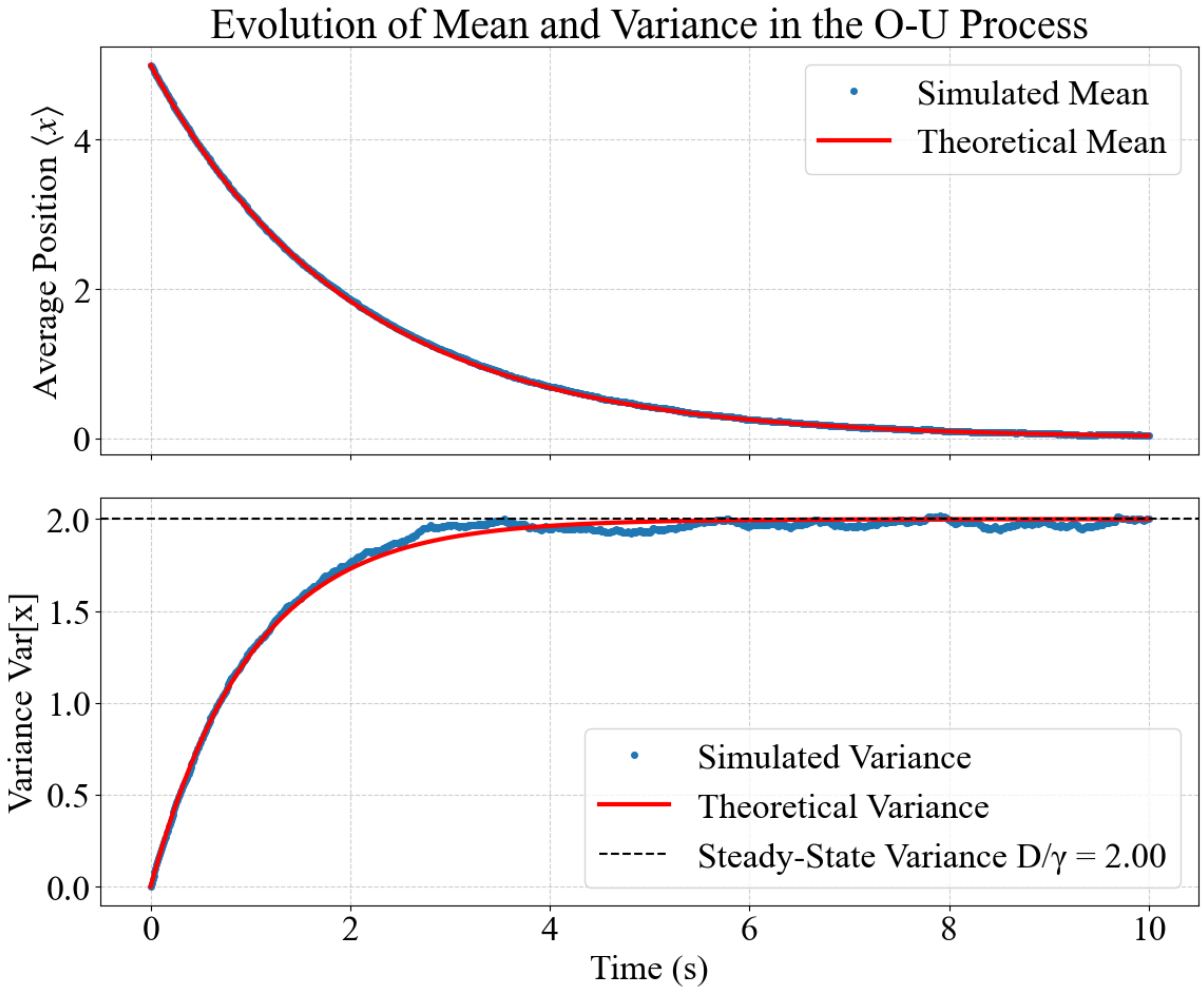

Figure 4:

This result is derived entirely from the perspective of stochastic dynamics (i.e., the Fokker-Planck equation).

Now, we temporarily put aside the stochastic process and turn to examine this system in thermal equilibrium from the perspective of classical statistical mechanics. The **Equipartition Theorem** is a fundamental conclusion of statistical mechanics (13). This theorem states that, in a system in thermal equilibrium at temperature T, every independent quadratic degree of freedom in the energy expression (such as the kinetic energy term $\frac{1}{2}m\mathbf{v}^2$ or the potential energy term $\frac{1}{2}k\mathbf{x}^2$) has an average energy exactly equal to $\frac{1}{2}k_BT$, where k_B is the Boltzmann constant.

For the particle we are studying, which is confined by the harmonic oscillator potential $U(\mathbf{x}) = \frac{1}{2}k\mathbf{x}^2$, its potential energy constitutes one quadratic degree of freedom. Therefore, according to the Equipartition Theorem, its average potential energy must satisfy:

$$\langle U \rangle = \left\langle \frac{1}{2}k\mathbf{x}^2 \right\rangle = \frac{1}{2}k\langle \mathbf{x}^2 \rangle = \frac{1}{2}k_BT \tag{24}$$

From this purely **thermodynamic relation**, we can directly solve for the mean squared value of the particle's position in thermal equilibrium:

$$\langle \mathbf{x}^2 \rangle_{\text{eq}} = \frac{k_BT}{k} \tag{25}$$

Physically, this steady state in the long-time limit corresponds precisely to the system reaching a state of **thermal equilibrium** with its surrounding environment (heat bath).

We now have two expressions for the same physical quantity (the mean squared position $\langle \mathbf{x}^2 \rangle$ in the steady state) derived from two completely different theoretical frameworks:

1. **From Stochastic Dynamics (FPE):** $\langle \mathbf{x}^2 \rangle_{\text{ss}} = \frac{D}{\gamma}$
2. **From Equilibrium Statistical Mechanics (Equipartition):** $\langle \mathbf{x}^2 \rangle_{\text{eq}} = \frac{k_B T}{k}$

Since these two expressions describe the same physical state, they must necessarily be equal:

$$\frac{D}{\gamma} = \frac{k_B T}{k} \tag{26}$$

Now, we substitute the relaxation rate $\gamma = k/\zeta = k/(6\pi\eta R)$ that we defined earlier:

$$\frac{D}{k/(6\pi\eta R)} = \frac{k_B T}{k} \quad \Longrightarrow \quad \frac{k}{6\pi\eta R} \cdot D = k_B T \tag{27}$$

Performing simple algebraic rearrangement on this equation, the friction constant k on both sides conveniently cancels out, and we obtain the famous **Einstein-Stokes Relation**:

$$D = \frac{k_B T}{6\pi\eta R} = \frac{k_B T}{\zeta} \tag{28}$$

The elegance of this formula goes beyond merely connecting several physical quantities; it lies in the profound physical principle it reveals, which is a specific instance of what is known as the **Fluctuation-Dissipation Theorem (FDT)**. Let us interpret both sides of this relation:

- **Fluctuation:** The diffusion coefficient D on the left side of the equation describes the intensity of the intrinsic, microscopic random fluctuations within the system. It quantifies the severity of the "kicks" or "jiggles" resulting from the incessant random collisions between the particle and the environmental molecules. This is a measure of the system's "disorder" and "randomness."
- **Dissipation:** The denominator on the right side of the equation, the friction coefficient $\zeta = 6\pi\eta R$, describes a macroscopic, deterministic energy dissipation process. When we try to push a particle through a fluid, the fluid creates resistance (drag). This resistance causes the macroscopic kinetic energy of the particle to be dissipated into heat. This is a measure of the system's "damping" and "friction."

The Einstein-Stokes relation strikingly points out that **Fluctuation and Dissipation are not independent phenomena; they are two different sides of the same physical process.** This process arises from the same microscopic origin—the random interactions between the system and its environment (the heat bath), where the constant microscopic collisions that cause drag on a macroscopically moving object are precisely the random forces that drive the erratic movement (Brownian motion) of the microscopic particle.

Temperature T plays a crucial role in this relation, acting like a bridge that links microscopic random fluctuations to macroscopic energy dissipation. A hotter environment implies more intense thermal motion of its constituent particles. This leads both to stronger random "kicks" on the Brownian particle (a larger D) and to greater resistance (a larger η, leading to a larger ζ) experienced by a macroscopic object attempting to move through the fluid. The Fluctuation-Dissipation Theorem precisely quantifies the relationship between these two effects.

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