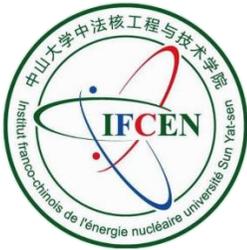


# 基于双核模型 对多核子转移反应角分布的研究

中国原子能科学研究院·多核子转移反应研讨会



报告人：廖泽鸿

导师：祝龙

合作者：苏军，高泽鹏，杨育

# 目录

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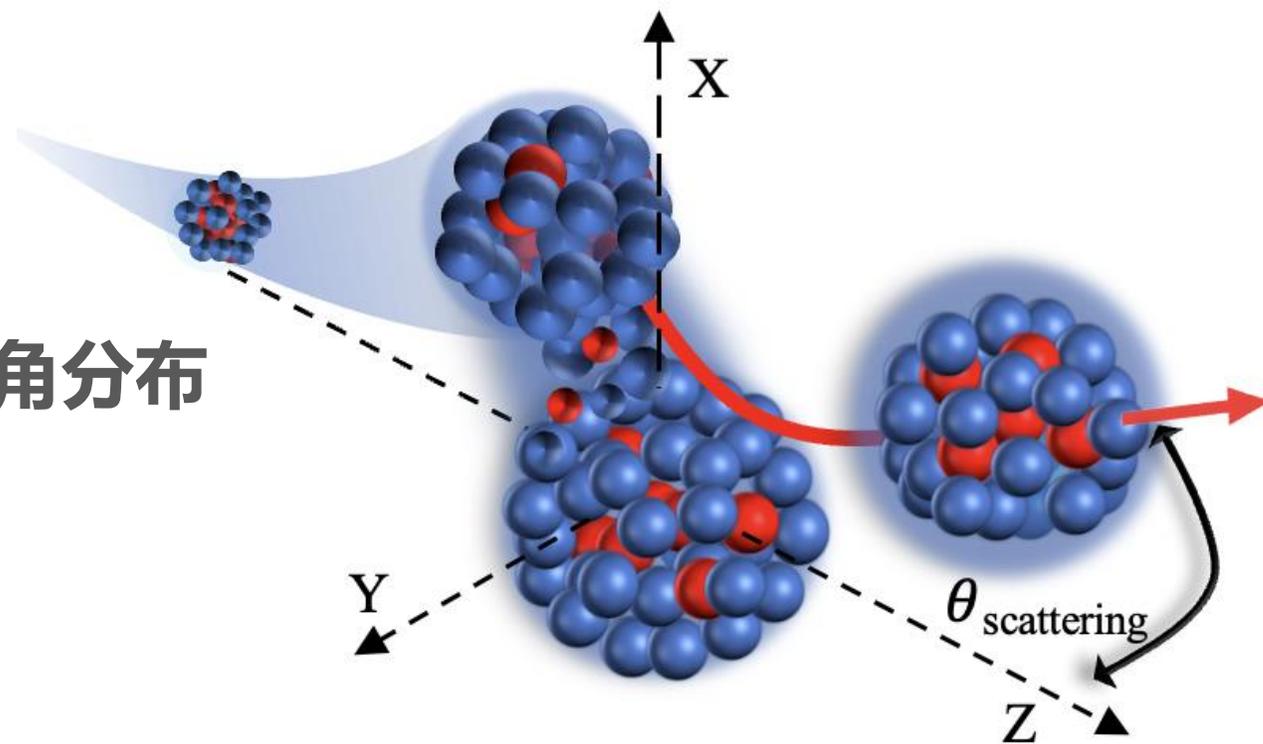
引言

2

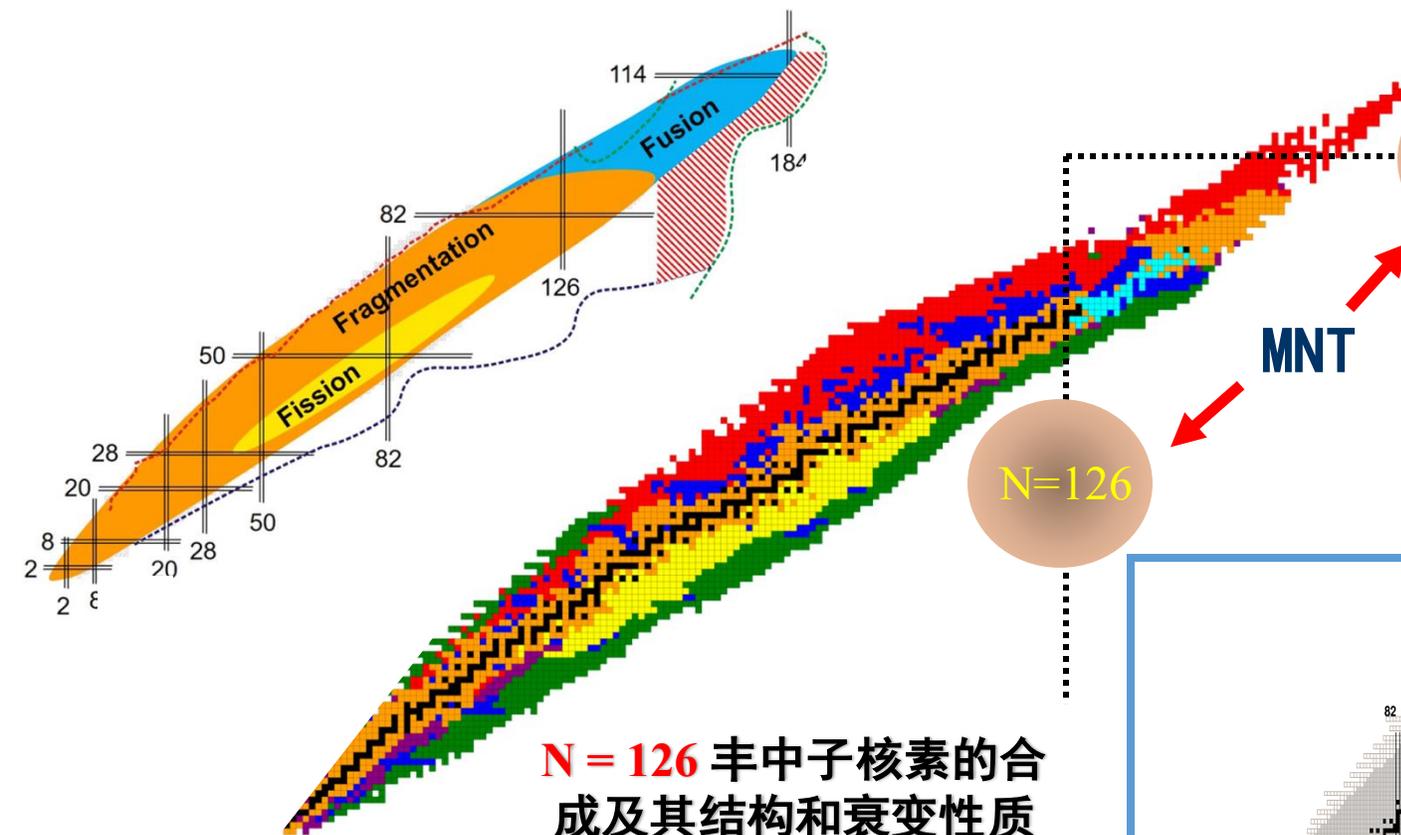
多核子转移反应的角分布

3

总结与展望



Zagrebaev, W. Greiner / Nuclear Physics A 944 (2015) 257–307



超重稳定岛

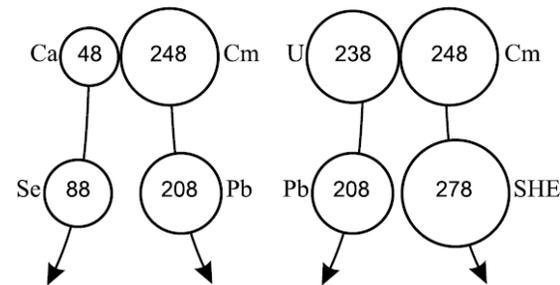
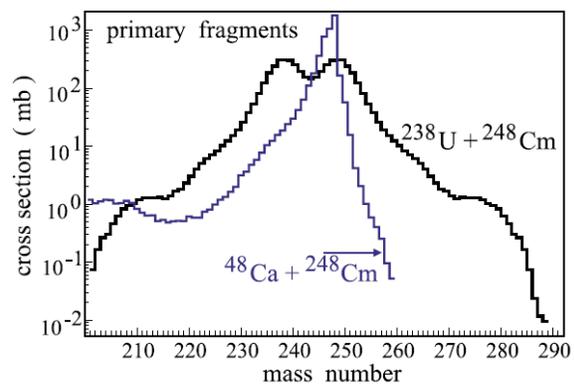
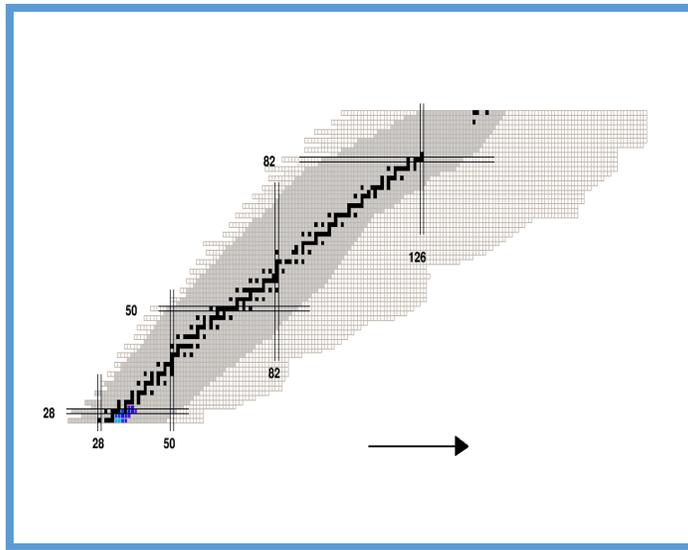
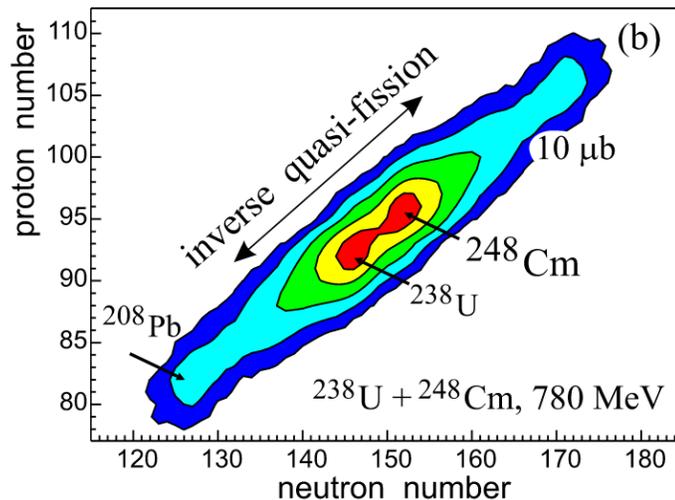
MNT

N=126

**N = 126 丰中子核素的合成及其结构和衰变性质**

<https://people.nsl.msui.edu/~thoennes/isotopes/>

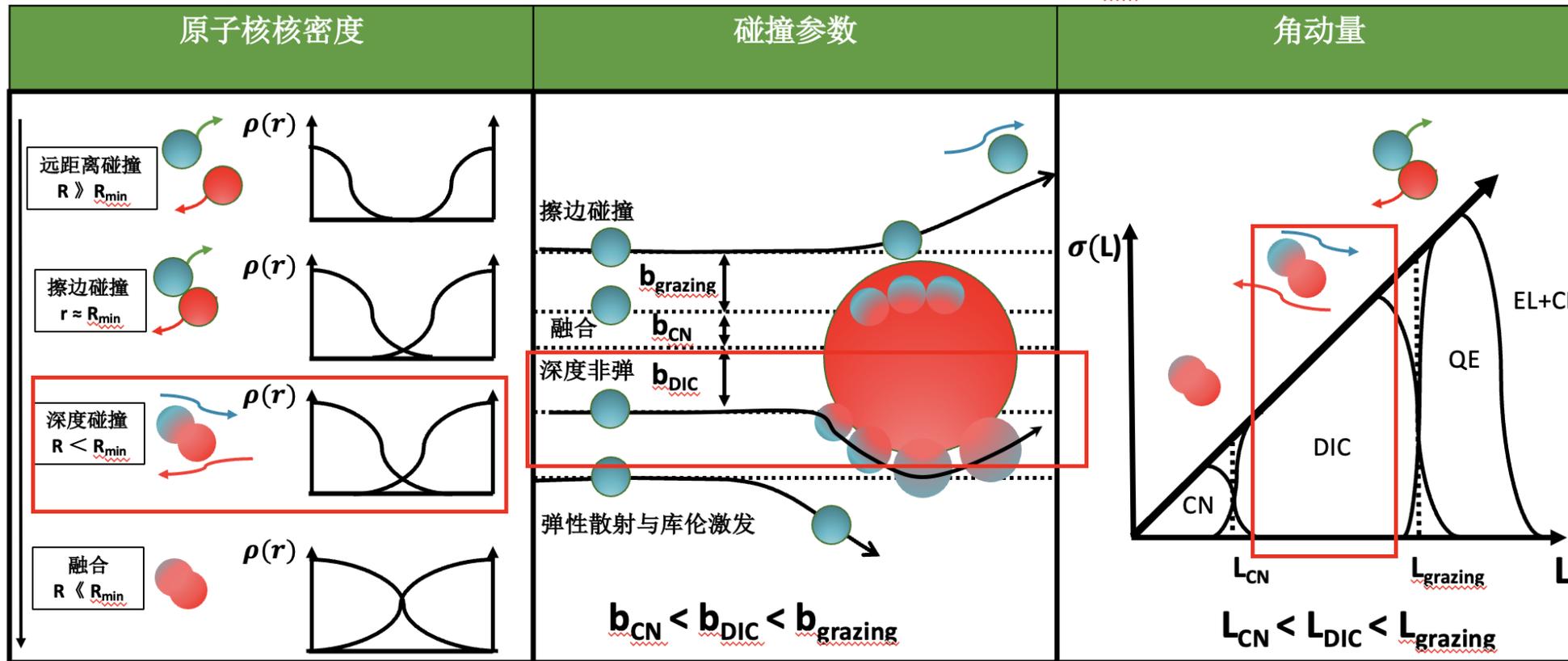
核天体物理中 **r** 过程的具体天体环境和详细演化路径。



产生丰中子超重核素非常有前景（也许是目前唯一）的方法

## Scattering types in heavy ion collision:

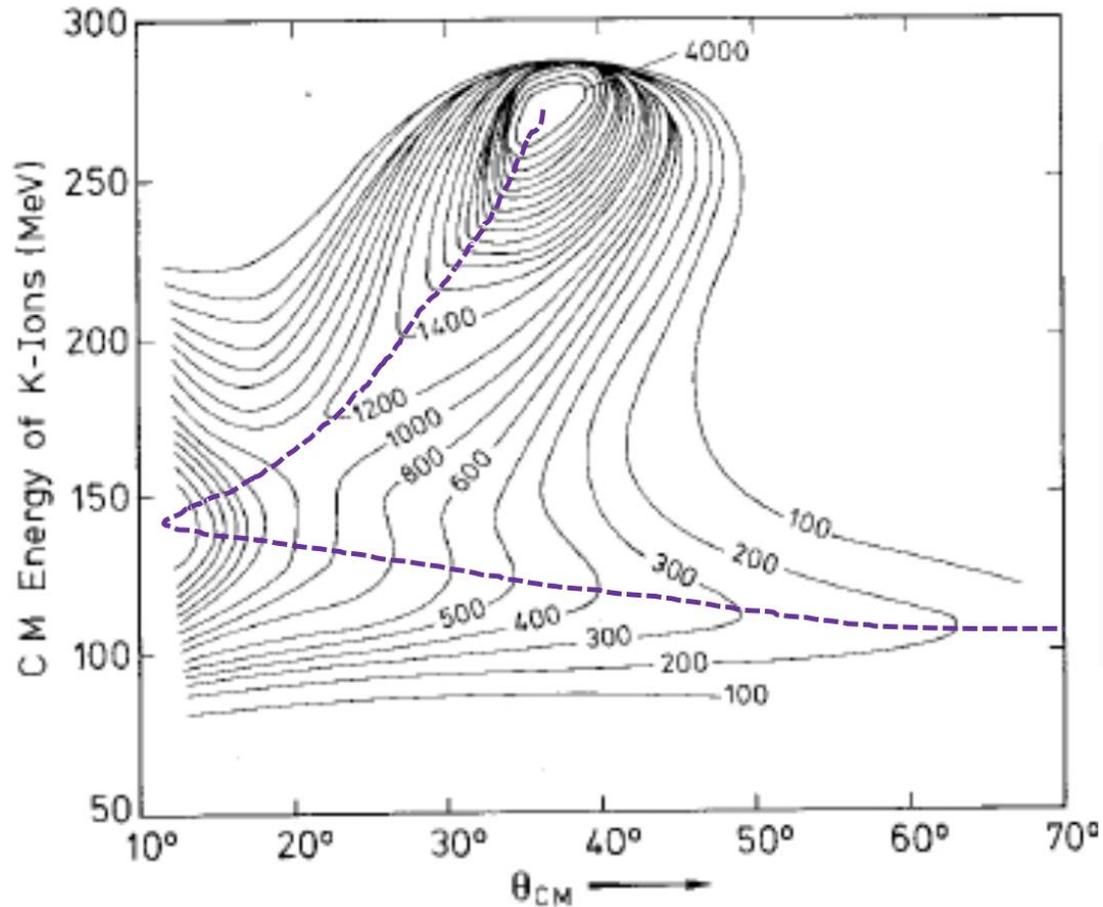
$E_{lab} < 10 \text{ MeV/u}$



融合反应: CN 深度非弹反应: DIC 准弹性反应: QE 擦边反应: Grazing

不同反应类型 角分布特征也不一样

$^{40}\text{Ar}+^{232}\text{Th}$  9.48 MeV/u



## ➤ 多核子转移反应:

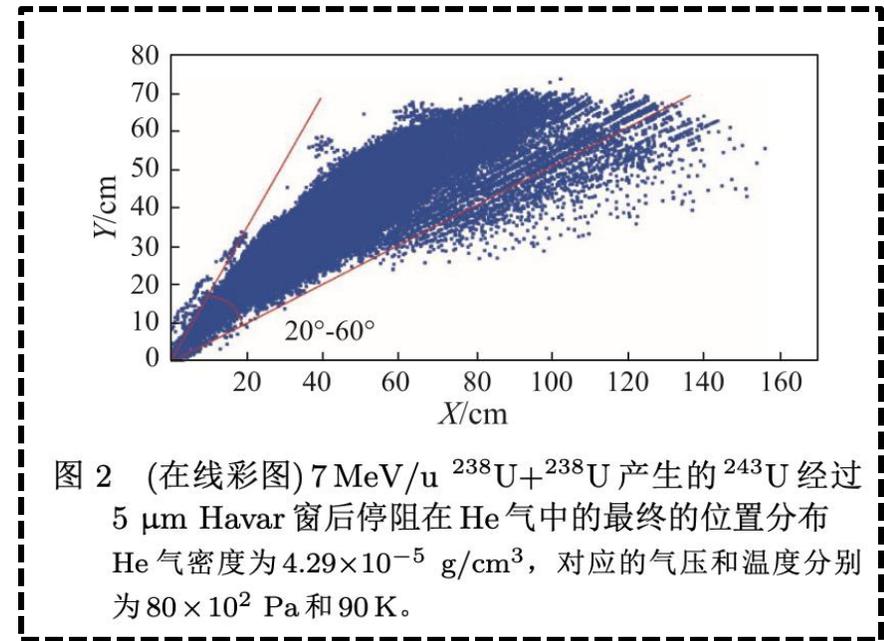
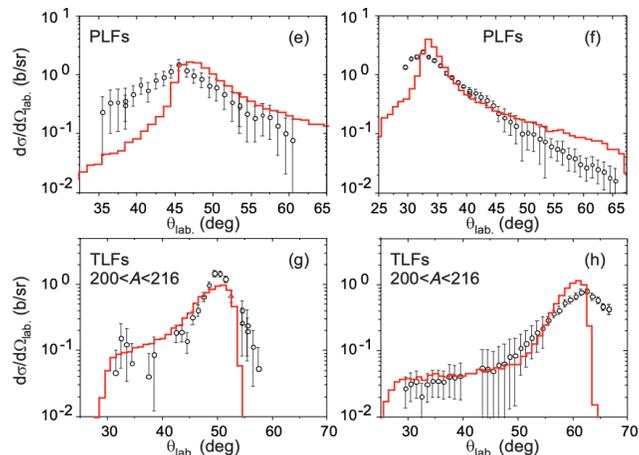


图 2 (在线彩图) 7 MeV/u  $^{238}\text{U}+^{238}\text{U}$  产生的  $^{243}\text{U}$  经过 5  $\mu\text{m}$  Havar 窗后停阻在 He 气中的最终的位置分布 He 气密度为  $4.29 \times 10^{-5}$  g/cm<sup>3</sup>, 对应的气压和温度分别为  $80 \times 10^2$  Pa 和 90 K。

黄文学等. HIAF上基于多核子转移反应的综合谱仪的机遇与挑战 [J]. 原子核物理评论, 2017, 34(3): 409-413.

- MNT反应产物的出射在实验室系并不是  $0^\circ$  附近的前冲方向, 而是覆盖了一个大范围的圆锥角
- 这给收集和分离我们感兴趣的多核子转移反应产物带来了很大的困难, 需要理论支持。

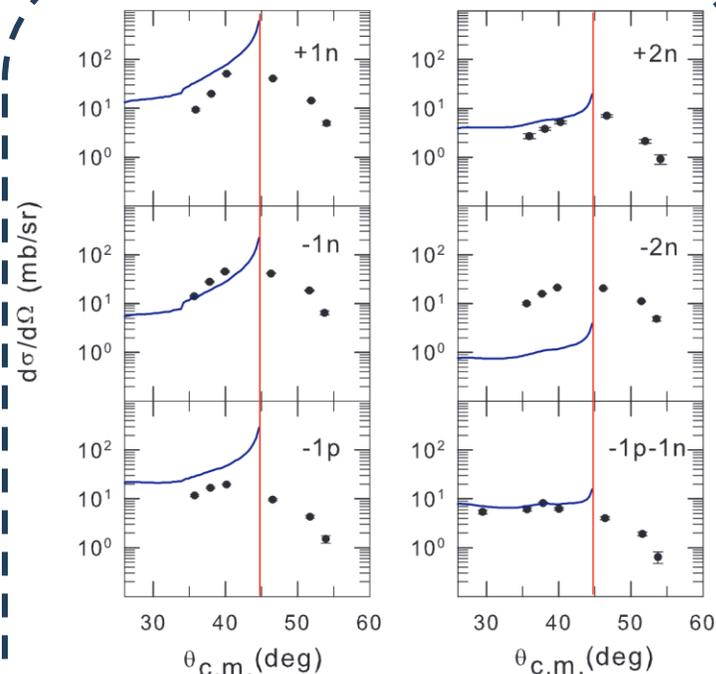
## ➤ 多维朗之万方程:



$$\dot{q}_i = \mu_{ij} p_j, \quad \dot{p}_i = F_i^{\text{driving}} + F_i^{\text{friction}} + F_i^{\text{random}}$$

A. V. KARPOV AND V. V. SAIKO, *PHYSICAL REVIEW C* 96, 024618 (2017)

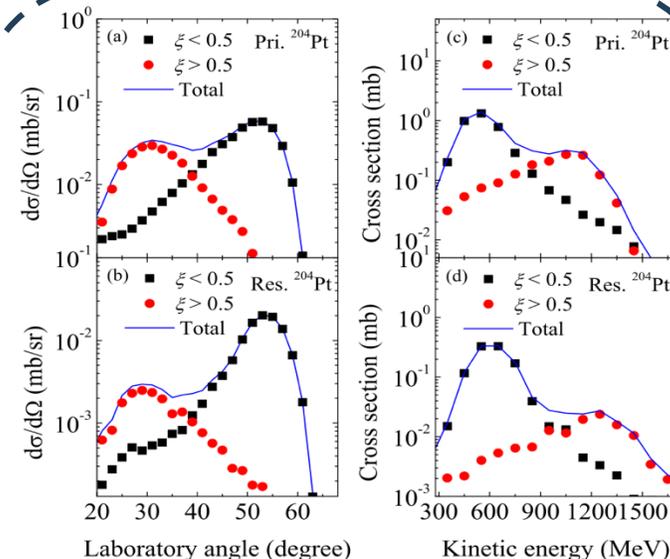
## ➤ 时间相关的Hartree Fock:



$$\frac{d\sigma(\Theta, N, Z)}{d\Omega} = \sum_i \frac{b_i}{\sin \Theta} \left| \frac{d\Theta(b_i)}{db} \right|^{-1} P_{\text{tr}}(b_i, N, Z),$$

K. Sekizawa. et,al, *PHYSICAL REVIEW C* 92, Q24603 (2015)

## ➤ 分子动力学模型:

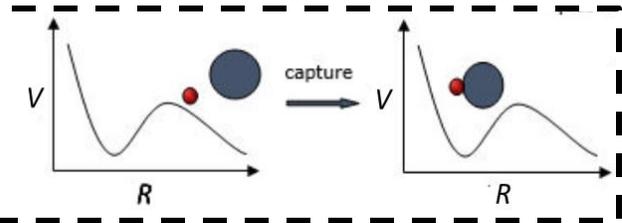


$$\dot{\vec{r}}_i = \frac{\partial H}{\partial \vec{p}_i}, \quad \dot{\vec{p}}_i = -\frac{\partial H}{\partial \vec{r}_i}$$

K. Zhao, et, al, *PHYSICAL REVIEW C* 106, L011602 (2022)

目前, 对于双核模型, 尚不能提供合理的角分布微分截面

Zhang, H-F., et al., *PROGRESS IN PHYSICS*, Vol.41 No.4



## 一维主方程 描述融合

Li, W.F., et al.,  
*Europhys. Lett.*,  
2003, 64(6): 750

## 二维主方程 区分中子质子 描述MNT

Feng, Z.-Q., et al.,  
*Nucl Phys A*, 2009  
816: 33

## 多维主方程 考虑形变组态

Zhu, L., *Phys Lett B*, 2021.  
816: 136226

Bao, X.J. *Phys Lett B*, 2022.  
833: 137307.

## 广义双核模型? 高维主方程?

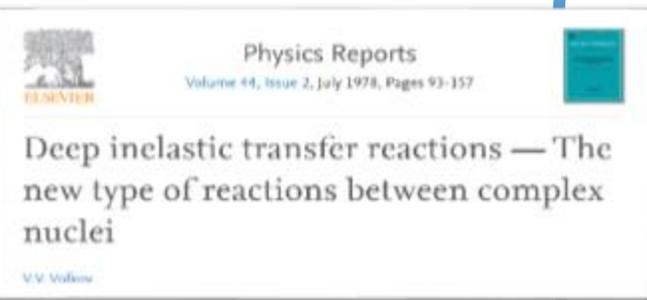
Zhu, L, to be published

Couple master and Langevin

More improvement  
.....  
.....

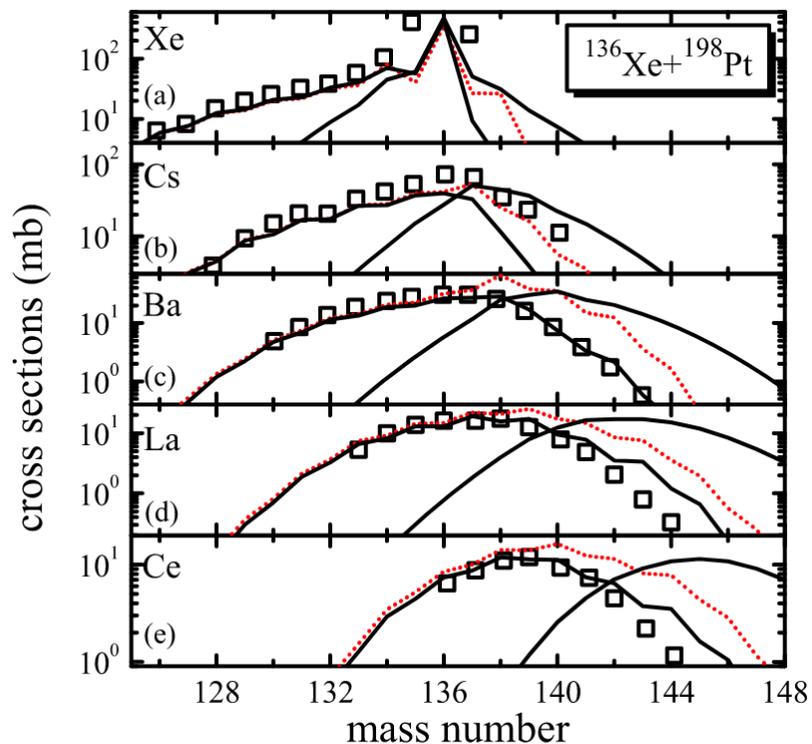
Dy-DNS

Wen, P.W., to be published



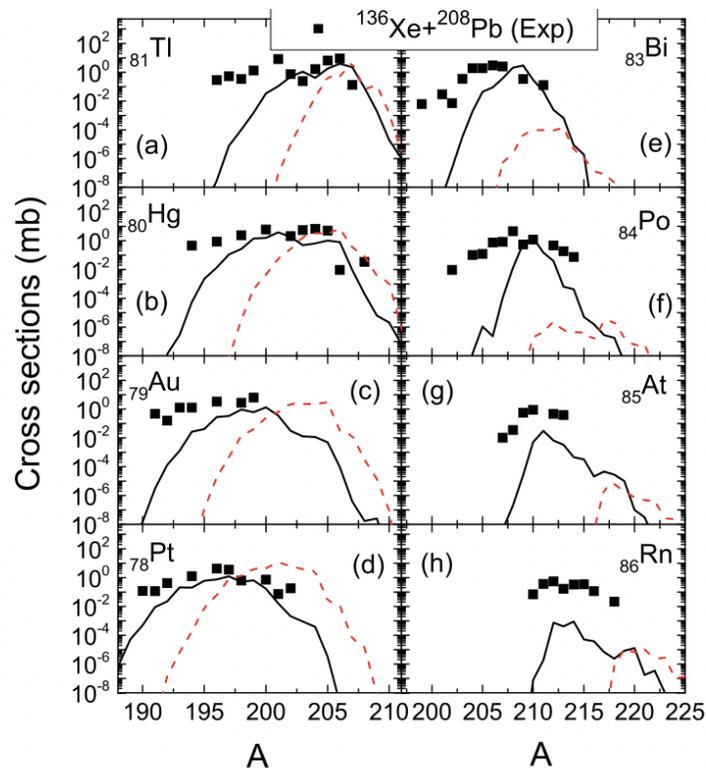
Adamian, et al.,  
*Phys Rev C*, 2005.  
71: 034603.

□  $^{136}\text{Xe}+^{198}\text{Pt}$ :



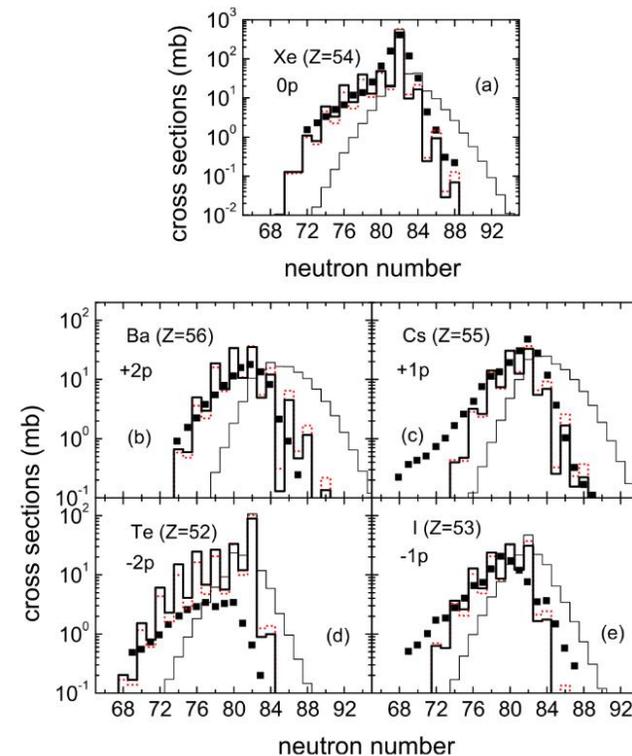
*Physics Letters B* 816 (2021) 136226

□  $^{136}\text{Xe}+^{208}\text{Pb}$



*Physics Letters B* 767 (2017) 437–442

□  $^{136}\text{Xe}+^{238}\text{U}$



*J. Phys. G: Nucl. Part. Phys.* 47 065107

基于双核模型，需要进一步构建合理的角分布微分截面

过去, 可以使用偏转函数去估计碎片的出射角度

*Li, J.Q, et al, PhysRevC.27.590*

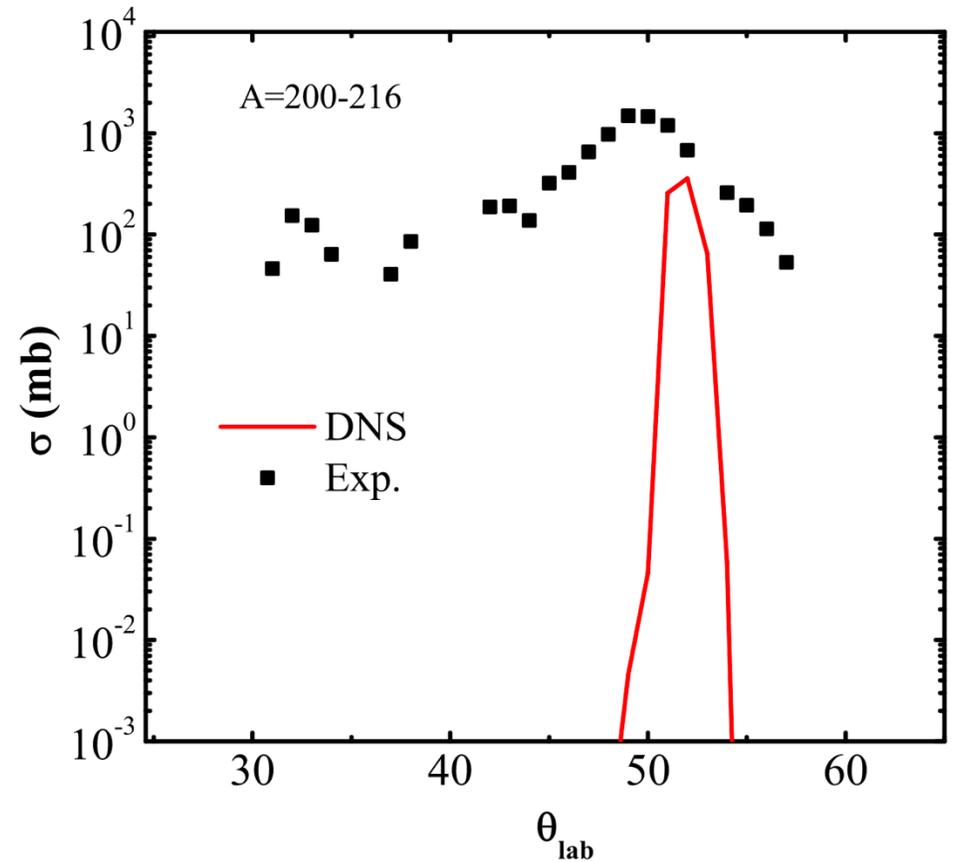
$$\Theta(J_i) = \underbrace{\Theta_C(J_i)}_{\text{库伦项}} + \underbrace{\Theta_N(J_i)}_{\text{核力项}}$$

$$\Theta(J_i)_C = 2 \arctan \frac{Z_p Z_t e^2}{2E_{c.m.} b} \quad \Theta(J_i)_N = -\beta \Theta_C^{gr}(J_i) \frac{J_i}{J_{gr}} \left( \frac{\delta}{\beta} \right)^{J_i/J_{gr}}$$

$$\beta = \begin{cases} 75f(\eta) + 15, & \eta < 375, \\ 36 \exp(-2.17 \times 10^{-3} \eta), & \eta \geq 375, \end{cases}$$

$$\delta = \begin{cases} 0.07f(\eta) + 0.11, & \eta < 375, \\ 0.117 \exp(-1.34 \times 10^{-4} \eta), & \eta \geq 375, \end{cases}$$

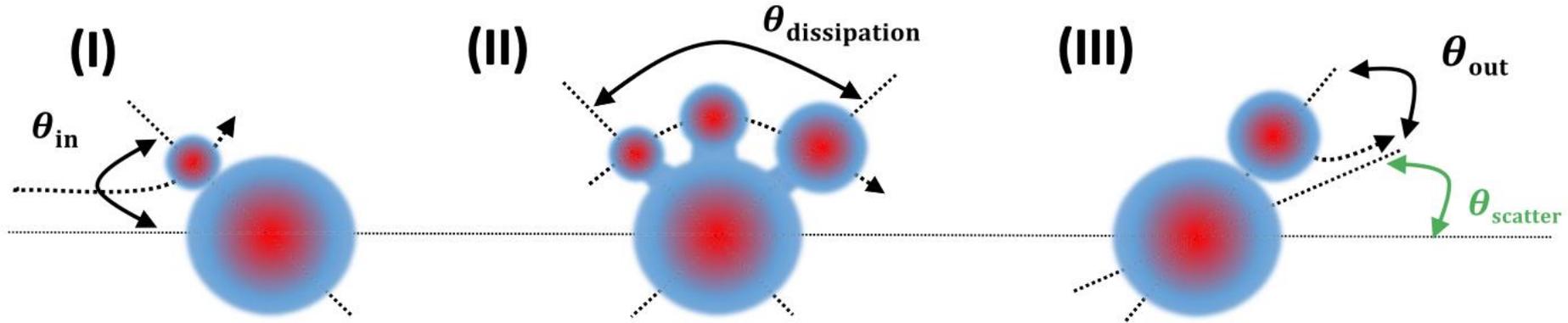
$$f(\eta) = \left[ 1 + \exp \frac{\eta - 235}{32} \right]^{-1}$$



*From Zhang, G, PHD paper*

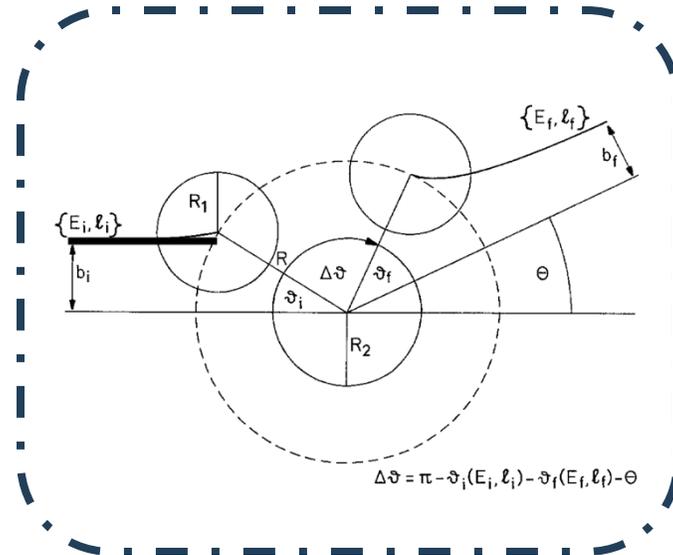
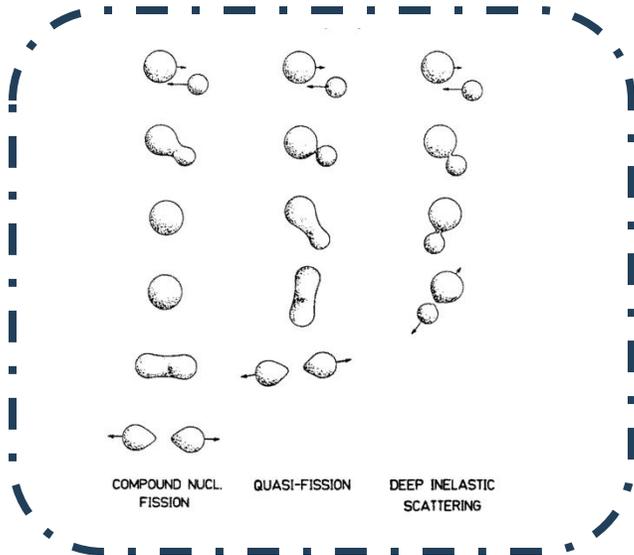
在峰位处, 计算值与实验值差距不大, 但是计算的角分布宽度太小, 低估了分布宽度值

PHYSICAL REVIEW RESEARCH 5, L022021 (2023)

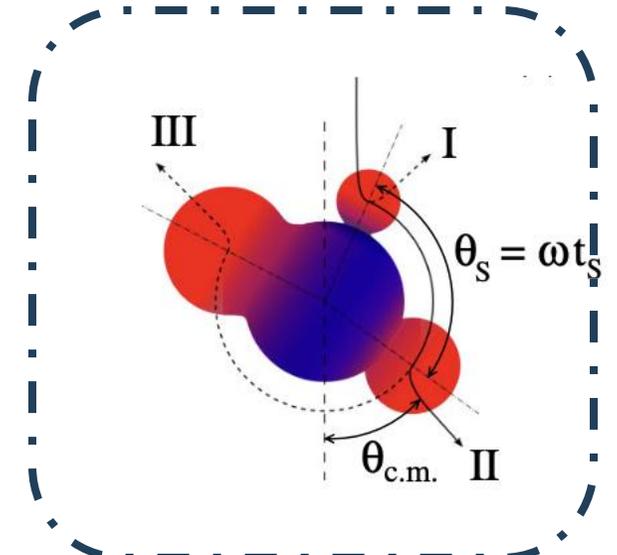


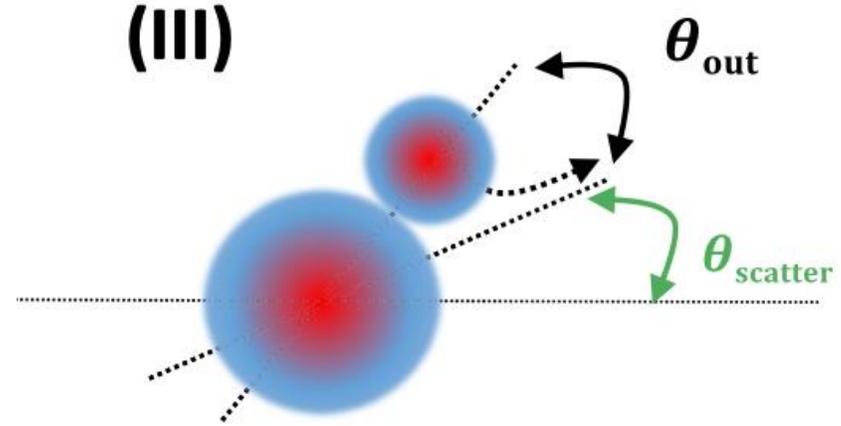
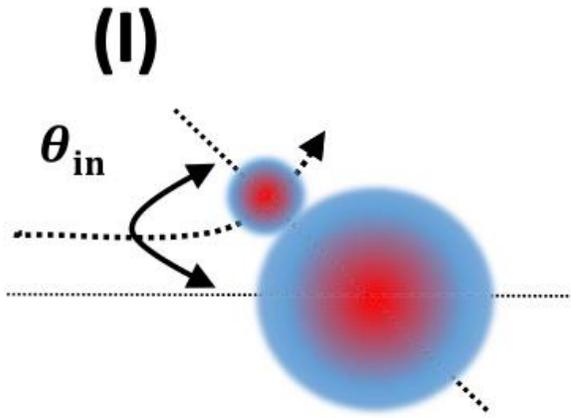
$$\theta_{\text{scatter}} = \pi - \theta_{\text{in}} - \theta_{\text{dissipation}} - \theta_{\text{out}},$$

◆ Under the Center of mass frame



$$\Delta\vartheta = \pi - \vartheta_i(E_i, l_i) - \vartheta_f(E_f, l_f) - \theta$$





PHYSICAL REVIEW RESEARCH 5, L022021 (2023)

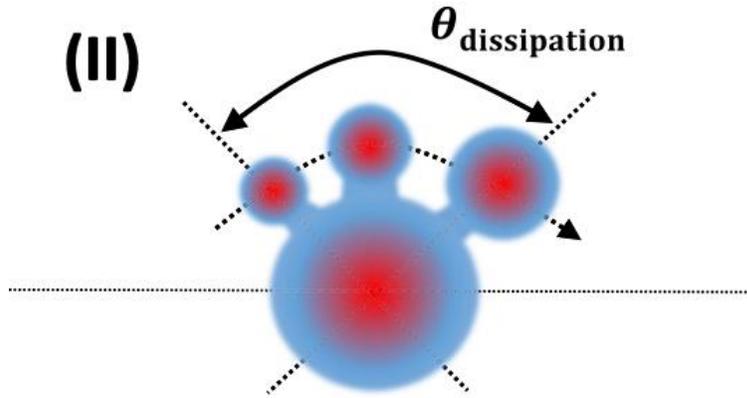
$\theta_{in}$  and  $\theta_{out}$  is determined by the Coulomb deflection and nuclear force effect

$$\mathcal{G} = \underbrace{\arcsin \frac{2b/R + \varepsilon}{\sqrt{4 + \varepsilon^2}}}_{\text{库伦项}} - \underbrace{\arcsin \frac{1}{\sqrt{(2/\varepsilon)^2 + 1}}}_{\text{核力项}}$$

G. Wolschin and W. Norenberg, *Z. Physik A* 284, 209-216 (1978)

$$\Theta = \pi - 2 \int_0^{b/r_{min}} \frac{dw}{\sqrt{1 - V(w)/E - w^2}} \quad (2.1.5)$$

Diagram illustrating the trajectory of a particle with impact parameter  $b$ , potential  $V(r)$ , and scattering angle  $\Theta(b)$ . The diagram shows the particle's path deflected by the potential, with the scattering angle  $\Theta(b)$  measured relative to the initial direction. The distance from the target to the point of closest approach is labeled  $r_{min}$ . The angle  $1/2(\pi - \Theta(b))$  is also indicated.

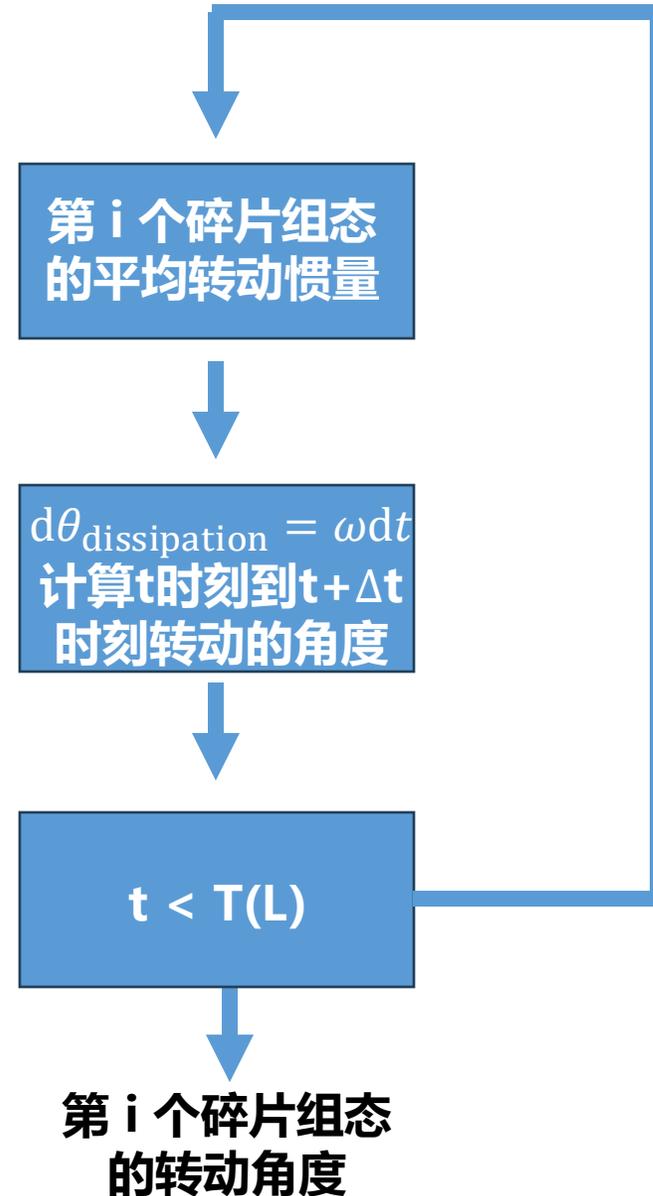


$\theta_{\text{dissipation}}$  由反应时间与转动速度决定

$$\theta_{\text{dissipation}} = \omega_{\text{DNS}} * t_s = \int^{t_s} \frac{J(t)\hbar}{\langle I \rangle} dt.$$

- 考虑角动量耗散的弛豫行为
- 考虑不同碎片组态的刚体转动惯量

PHYSICAL REVIEW RESEARCH 5, L022021 (2023)



$$U(Z_1, N_1, \beta_2, J, r = R_{\text{cont}}) = \Delta(Z_1, N_1) + \Delta(Z_2, N_2) + V(Z_1, N_1, \beta_2, J, r = R_{\text{cont}}) + \frac{1}{2}C_1(\delta\beta_2^1)^2 + \frac{1}{2}C_2(\delta\beta_2^2)^2.$$

$$\frac{dP(Z_1, N_1, \beta_2, J, t)}{dt}$$

$$= \sum_{Z'_1} W_{Z_1, N_1, \beta_2; Z'_1, N_1, \beta_2}(t) [d_{Z_1, N_1, \beta_2} P(Z'_1, N_1, \beta_2, J, t)$$

$$- d_{Z'_1, N_1, \beta_2} P(Z_1, N_1, \beta_2, J, t)]$$

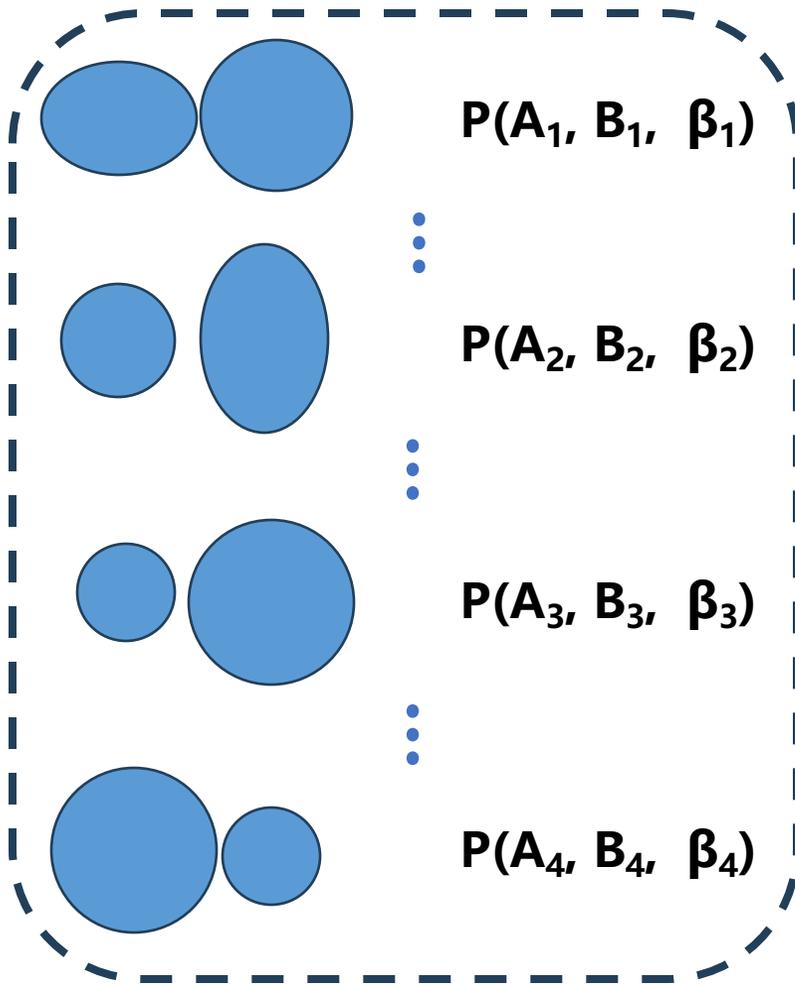
$$+ \sum_{N'_1} W_{Z_1, N_1, \beta_2; Z_1, N'_1, \beta_2}(t) [d_{Z_1, N_1, \beta_2} P(Z_1, N'_1, \beta_2, J, t)$$

$$- d_{Z_1, N'_1, \beta_2} P(Z_1, N_1, \beta_2, J, t)]$$

$$+ \sum_{\beta'_2} W_{Z_1, N_1, \beta_2; Z_1, N_1, \beta'_2}(t) [d_{Z_1, N_1, \beta_2} P(Z_1, N_1, \beta'_2, J, t)$$

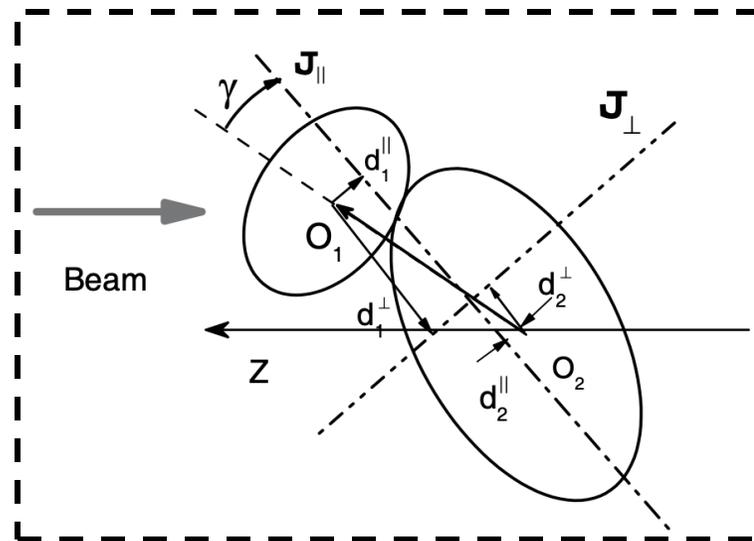
$$- d_{Z_1, N_1, \beta'_2} P(Z_1, N_1, \beta_2, J, t)].$$

(1)



不同碎片组态的概率

$$\langle I \rangle = \frac{1}{2} * (I_{\text{in}} + I_{\text{out}})$$

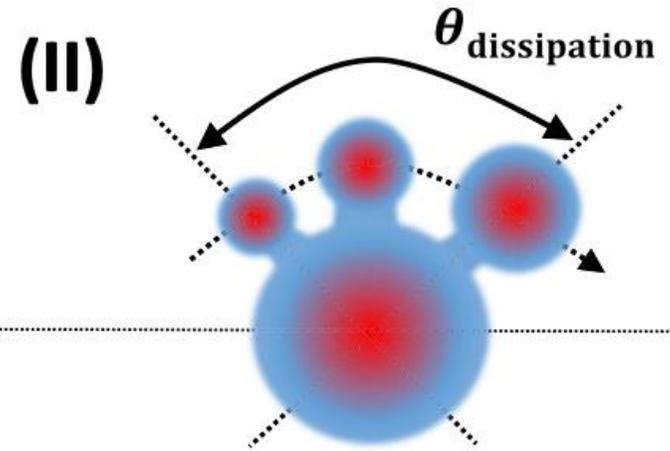
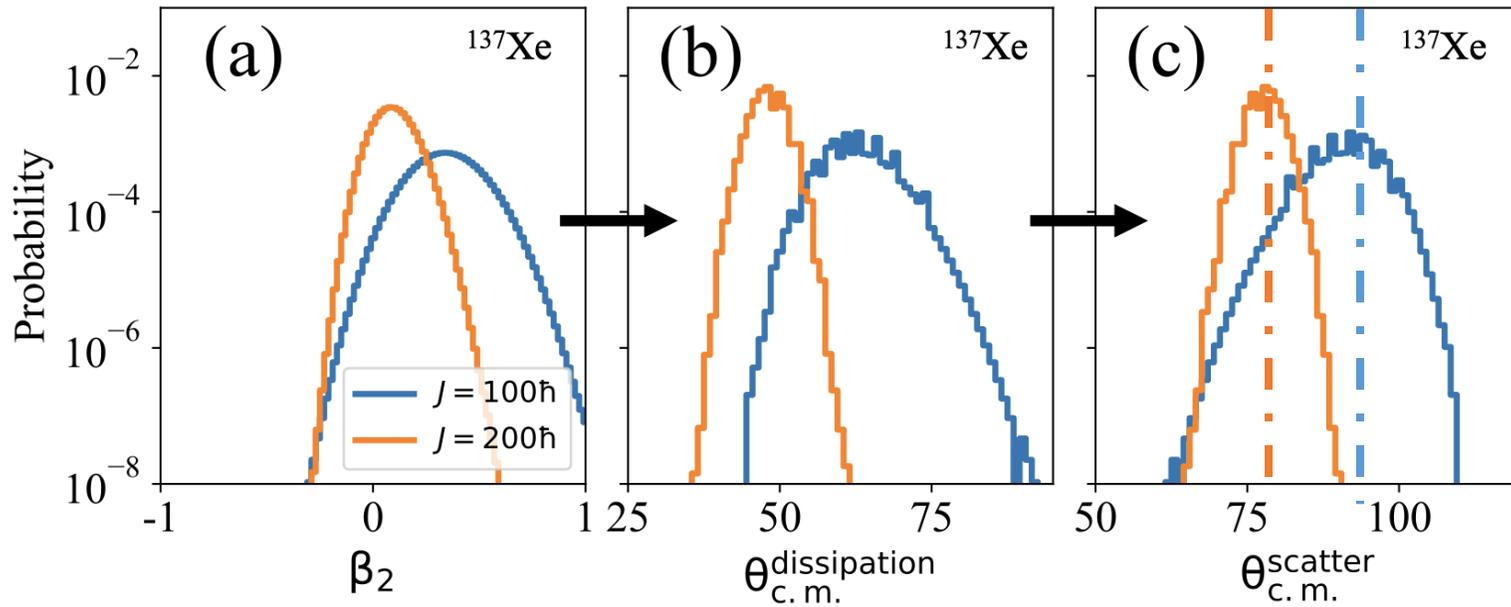


*Eur. Phys. J. A 34, 325–339 (2007)*

不同碎片组态的转动惯量



四级形变的概率  $\rightarrow$  耗散角的概率  $\rightarrow$  出射角的概率

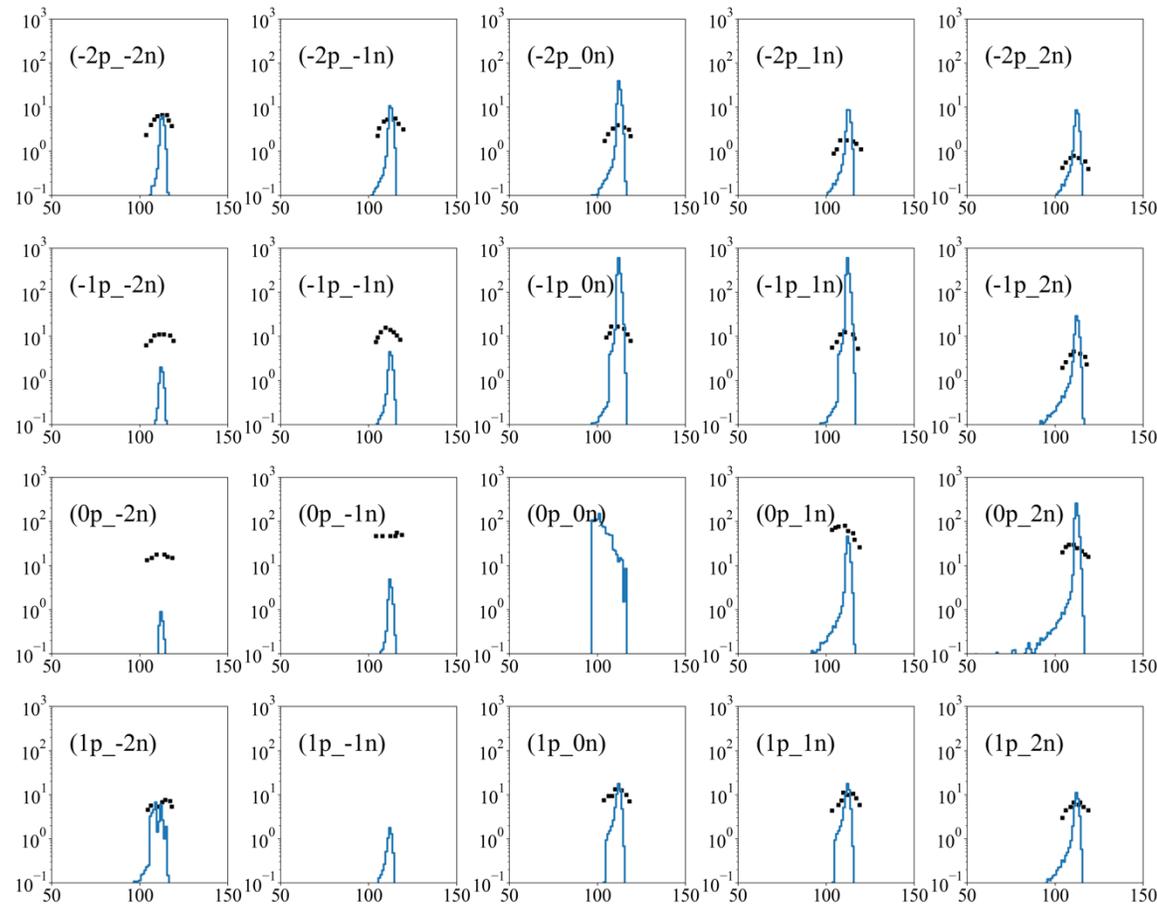


*PHYSICAL REVIEW RESEARCH 5, L022021 (2023)*

在双核模型中 碎片形变的自由度演化的引入 可以**自洽地**增加碎片出射角度的**涨落**

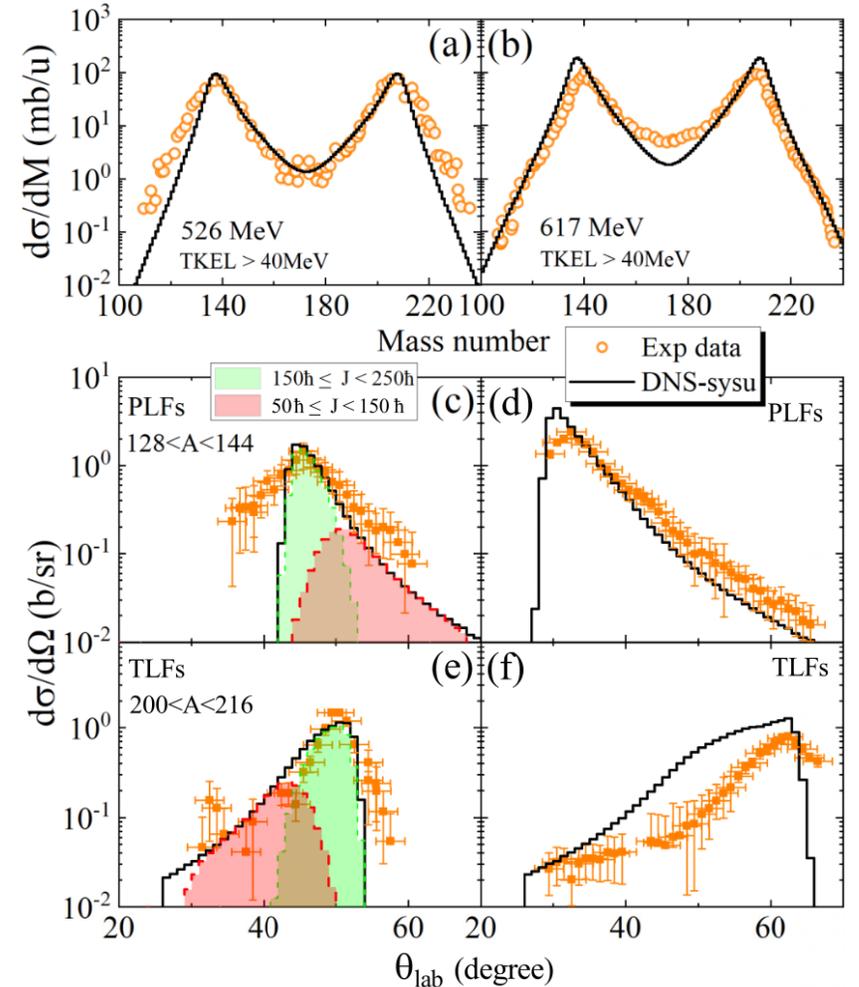
PHYSICAL REVIEW RESEARCH 5, L022021 (2023)

➤  $^{206}\text{Pb} + ^{118}\text{Sn}$  @  $E_{c.m.} = 437\text{MeV}$



Exp data from J. DIKLI' C et al. PHYSICAL REVIEW C 107(2023), 014619 (2023)

➤  $^{136}\text{Xe} + ^{208}\text{Pb}$  @  $E_{c.m.} = 526\text{MeV}$



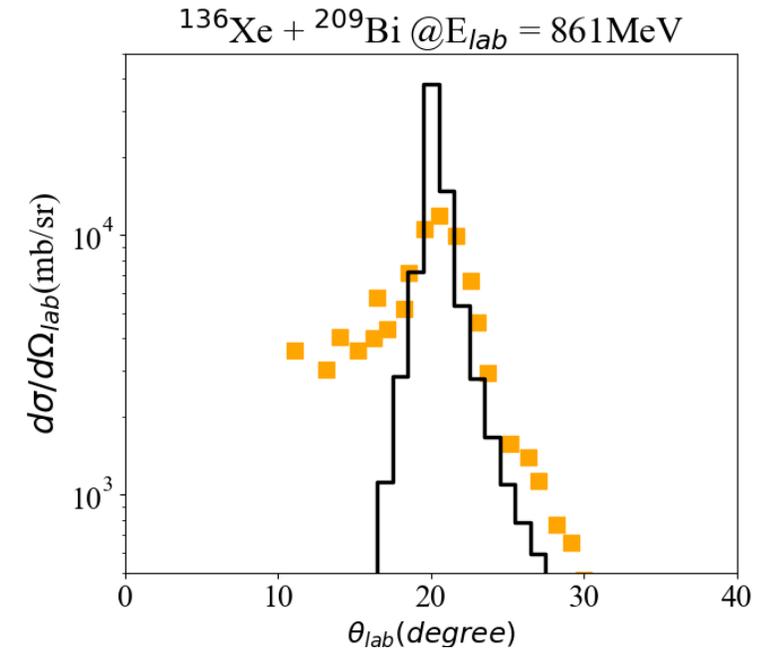
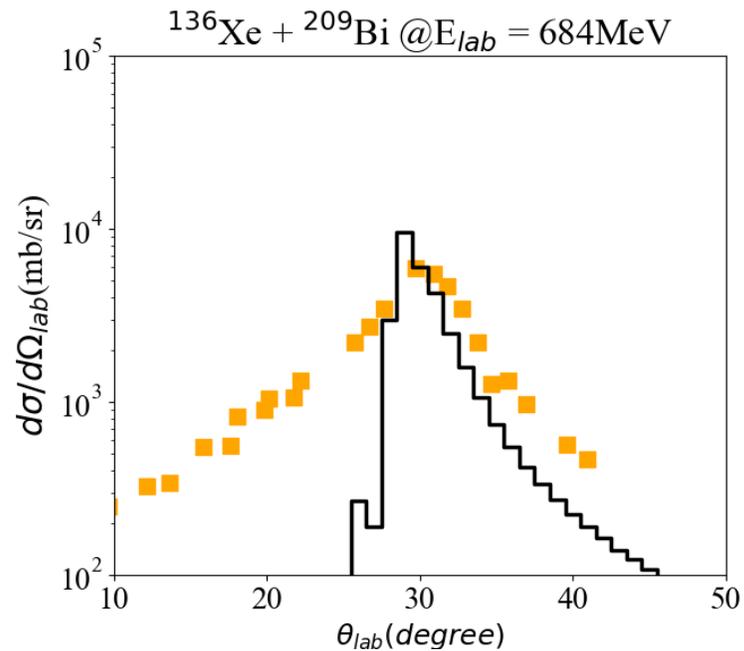
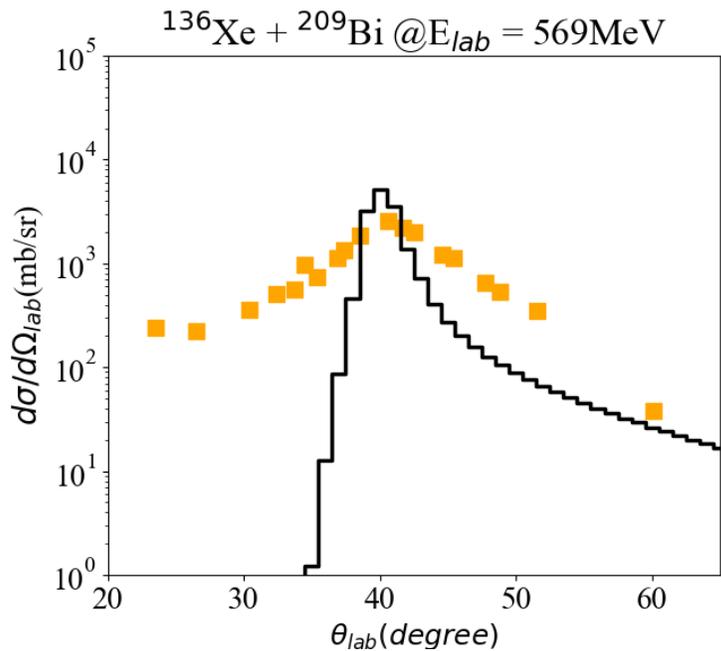
Exp data from E. M. Kozulin et al. Phys. Rev. C 86, 044611 (2012).

@  $E_{c.m.} = 569\text{MeV}$   
 $309 > \text{TKEL} > 23 \text{ MeV}$

@  $E_{c.m.} = 684\text{MeV}$   
 $384 > \text{TKEL} > 34 \text{ MeV}$

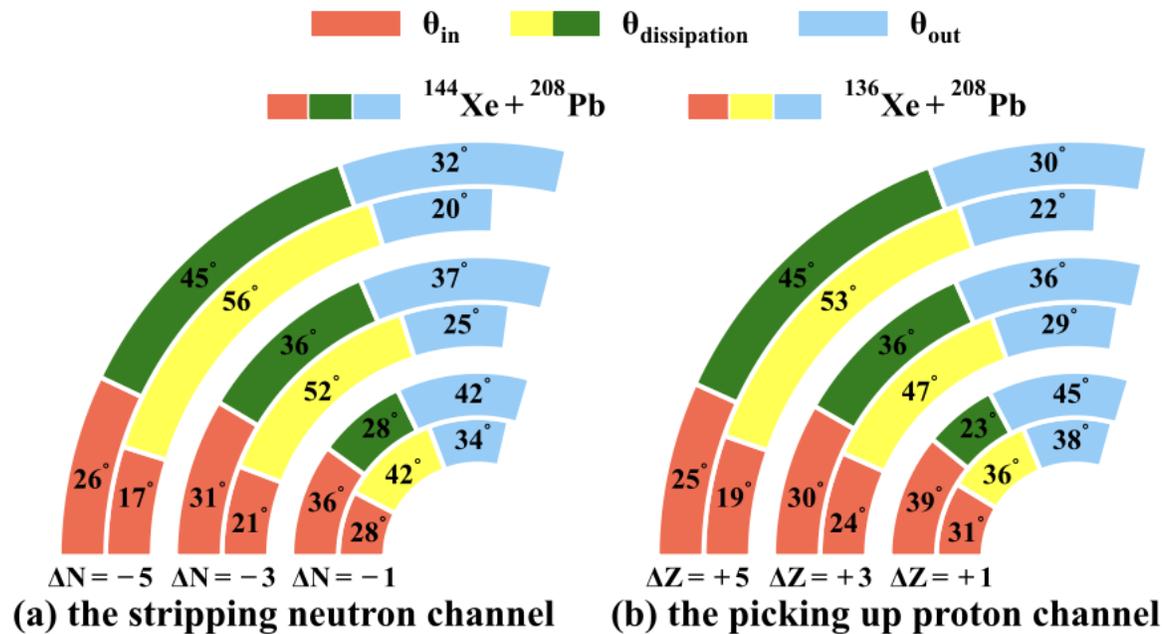
@  $E_{c.m.} = 861\text{MeV}$   
 $601 > \text{TKEL} > 51 \text{ MeV}$

➤ 实验系



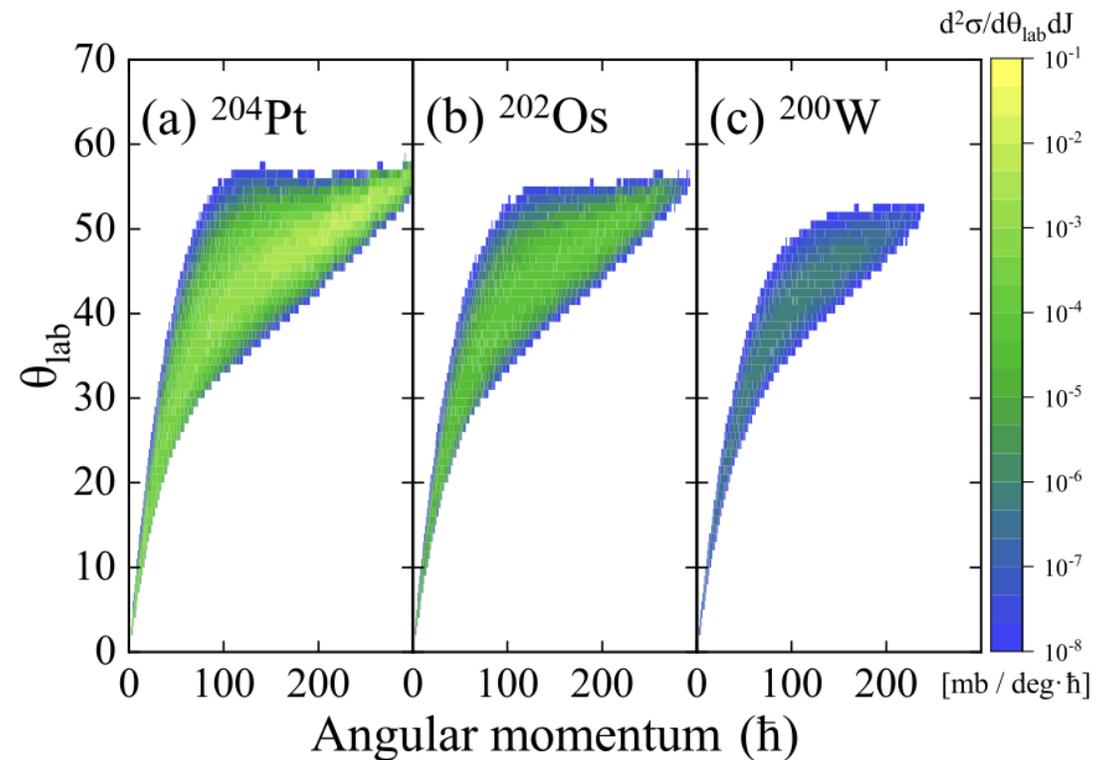
➤  $^{136}\text{Xe} + ^{208}\text{Pb}$  @  $E_{\text{c.m.}} = 526\text{MeV}$

➤ 不同出射碎片, 各部分角度的占比



- 库伦偏转角与耗散角的关系
- 纯中子、纯质子转移道

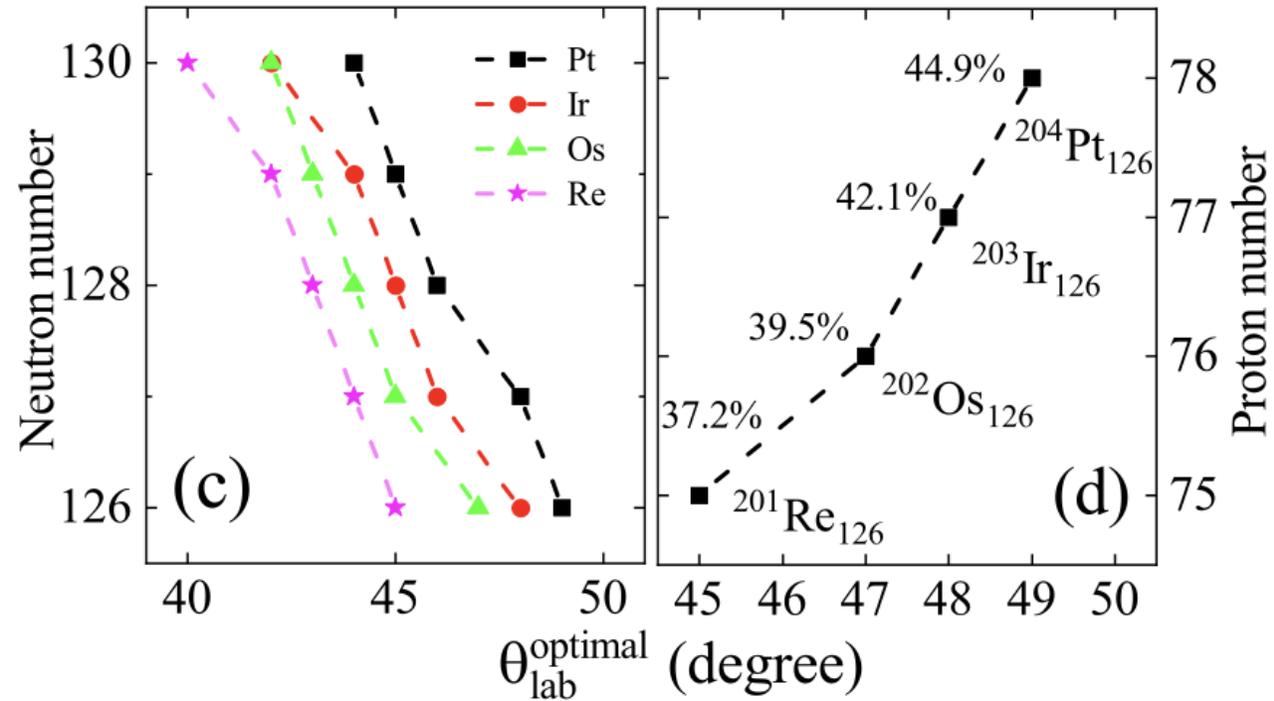
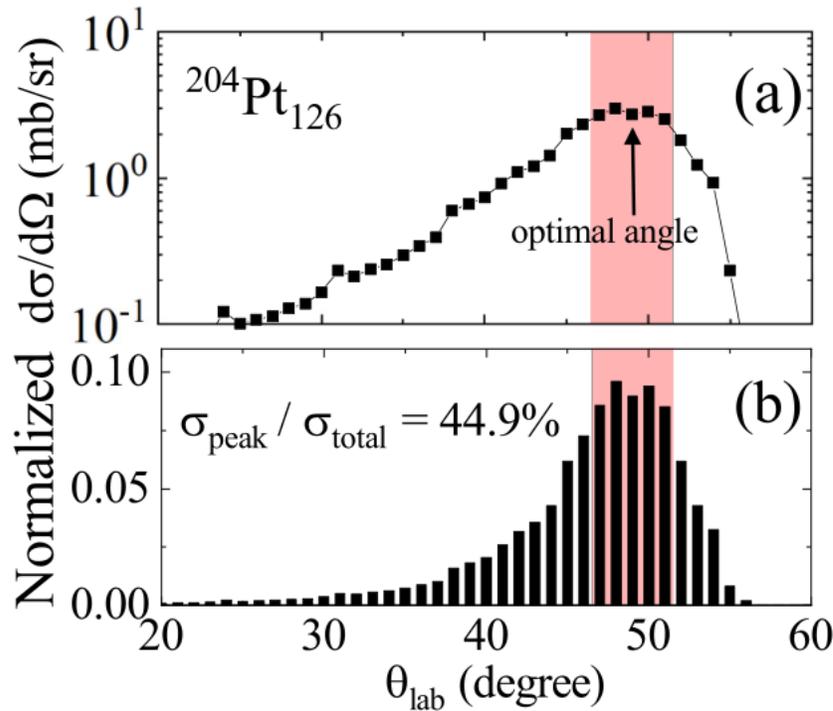
➤ N=126核素碎片的微分角分布截面



- 角动量依赖

$$\sum_{\theta-2}^{\theta+2} 2\pi \sin(\theta) \sigma(\theta)$$

➤  $^{136}\text{Xe} + ^{208}\text{Pb}$  @  $E_{\text{c.m.}} = 526\text{MeV}$



我们发现，在  $47^\circ < \theta_{\text{lab}} < 51^\circ$  的范围内可以检测到 44.9% 的产量比率。接近一半的产量可以在最佳角度周围的小范围角度中检测到。

## 总结:

- 本次工作，我们基于双核模型给出了合理的多核子转移反应的微分角分布。
- 我们从核素的微分角分布中，提取了最佳探测的角度，可供实验探测相关的预言。

过去, 可以使用偏转函数去估计碎片的出射角度

*Li, J.Q, et al, PhysRevC.27.590*

$$\Theta(J_i) = \underbrace{\Theta_C(J_i)}_{\text{库伦项}} + \underbrace{\Theta_N(J_i)}_{\text{核力项}}$$

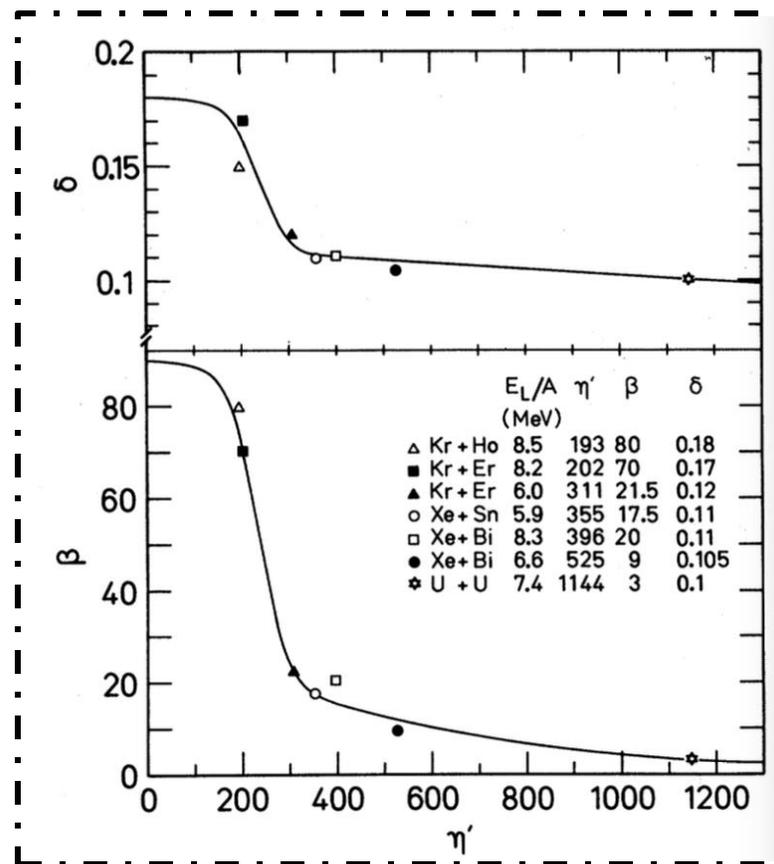
库伦项      核力项

$$\Theta(J_i)_C = 2 \arctan \frac{Z_p Z_t e^2}{2E_{c.m.} b} \quad \Theta(J_i)_N = -\beta \Theta_C^{gr}(J_i) \frac{J_i}{J_{gr}} \left( \frac{\delta}{\beta} \right)^{J_i/J_{gr}}$$

$$\beta = 75f(\eta) + 15, \quad \eta < 375, \\ 36 \exp(-2.17 \times 10^{-3} \eta), \quad \eta \geq 375,$$

$$\delta = 0.07f(\eta) + 0.11, \quad \eta < 375, \\ 0.117 \exp(-1.34 \times 10^{-4} \eta), \quad \eta \geq 375,$$

$$f(\eta) = \left[ 1 + \exp \frac{\eta - 235}{32} \right]^{-1}$$



展望:

- 更多的实验点?
- 更好的经验公式

## In this Work

Under the Center of mass frame

$$U(Z_1, N_1, \beta_2, J, r = R_{\text{cont}}) = \Delta(Z_1, N_1) + \Delta(Z_2, N_2) + V(Z_1, N_1, \beta_2, J, r = R_{\text{cont}}) + \frac{1}{2}C_1(\delta\beta_1^2) + \frac{1}{2}C_2(\delta\beta_2^2)$$

$$\frac{dP(Z_1, N_1, \beta_2, J, t)}{dt} = \sum_{Z_1'} W_{Z_1, N_1, \beta_2, Z_1', N_1', \beta_2'}(t) [d_{Z_1, N_1, \beta_2}(Z_1', N_1', \beta_2', J, t) - d_{Z_1', N_1', \beta_2'}(Z_1, N_1, \beta_2, J, t)] + \sum_{N_1'} W_{Z_1, N_1, \beta_2, Z_1, N_1', \beta_2'}(t) [d_{Z_1, N_1, \beta_2}(Z_1, N_1', \beta_2', J, t) - d_{Z_1, N_1', \beta_2'}(Z_1, N_1, \beta_2, J, t)] + \sum_{\beta_2'} W_{Z_1, N_1, \beta_2, Z_1, N_1, \beta_2'}(t) [d_{Z_1, N_1, \beta_2}(Z_1, N_1, \beta_2', J, t) - d_{Z_1, N_1, \beta_2'}(Z_1, N_1, \beta_2, J, t)] \quad (1)$$

不同碎片组态的概率

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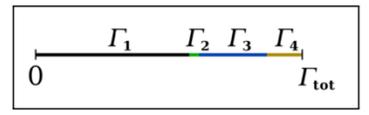
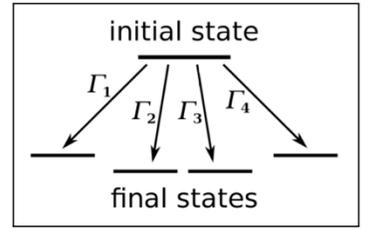
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## Dy-DNS

### 动力学蒙特卡罗方法 (dynamical Monte Carlo)

广义双核模型: 动力学双核模型 (DyDNS):

- 1 在时刻  $t$ , 求解转移主方程, 获取转移至不同态的概率  $P_{Z_i N_i}$
- 2 计算累积函数  $C_i = \sum_{j=1}^i P_{Z_j N_j}$ .
- 3 抽取随机数  $u \in (0, 1]$ ;
- 4 确定第  $i$  个态, 满足  $C_{i-1} < u \leq C_i$ ;
- 5 选取  $i$  作为终态, 求解  $t$  时刻到  $t + \Delta t$  时刻的相对运动方程;
- 6 重复第一步。



采用事件数模拟获取转移截面、角分布等信息。

Pineda, M. et al (2022). "Kinetic Monte Carlo simulations for heterogeneous catalysis: Fundamentals, current status, and challenges." J Chem Phys 156 120902

## 展望:

- 动力学? (广义双核模型)
- 更高维主方程求解? (狭义双核模型)



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**Thank you for your attention!**