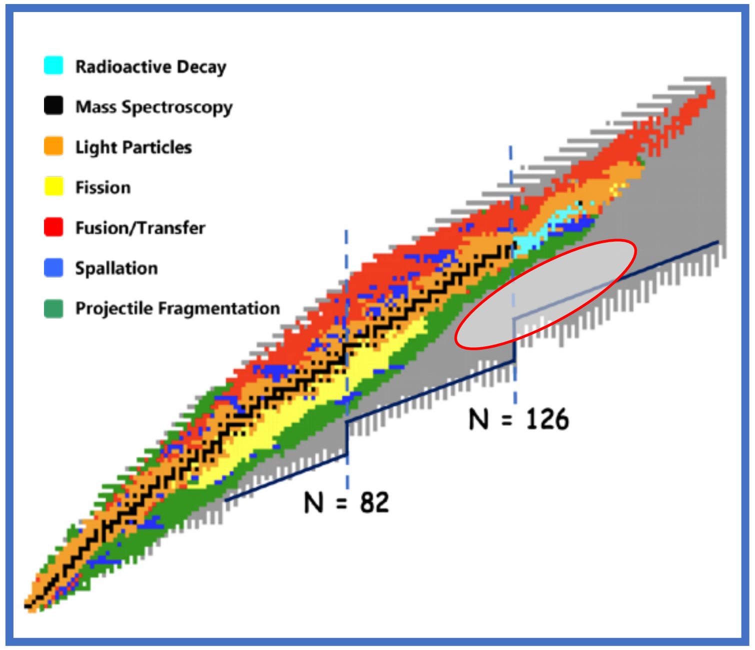
Shell effects on the drift and fluctuation in multinucleon transfer reactions

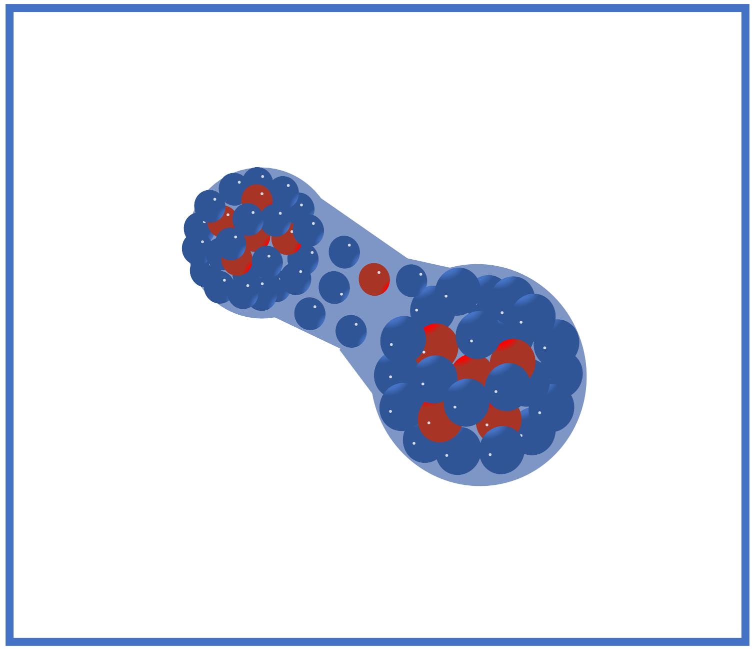
Sino-French Institute of Nuclear Engineering and Technology

Outlook

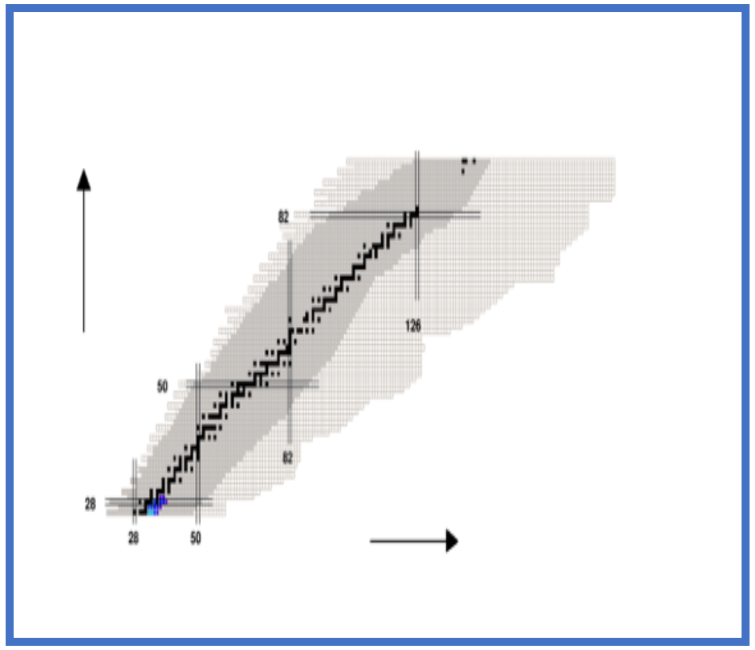
Background: Multinucleon transfer reaction



https://people.nscl.msu.edu/~thoennes/isotopes/



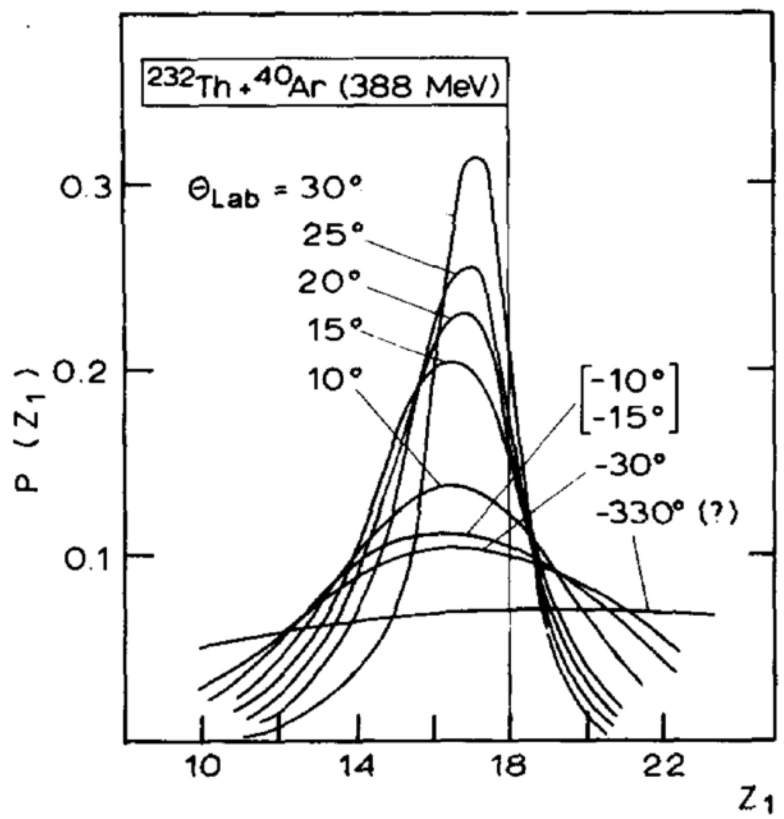
Phys. Rev. Lett. 115, 172503



Rep. Prog. Phys. 70, 1525 (2007).

How to extend the nuclide chart? → **Very promising way to produce neutron-rich superheavy nuclides** → **Specific astro-environments and detailed evolutionary pathways for r-processes in nuclear astrophysics**

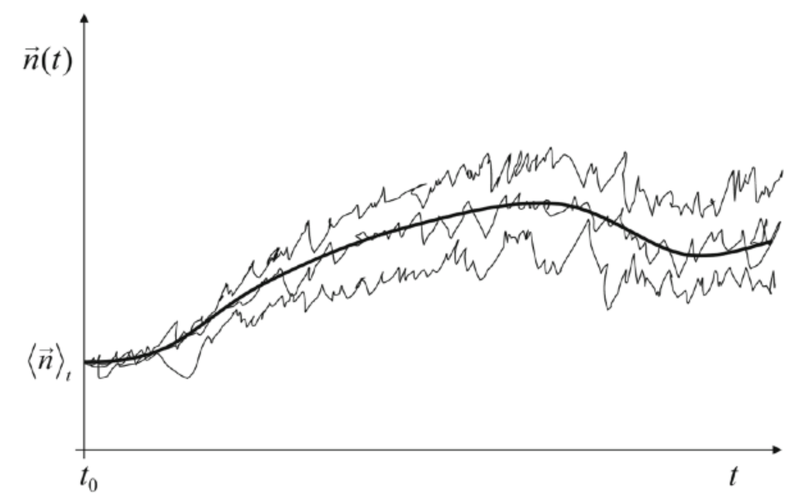
Transport phenomena in nuclear reactions



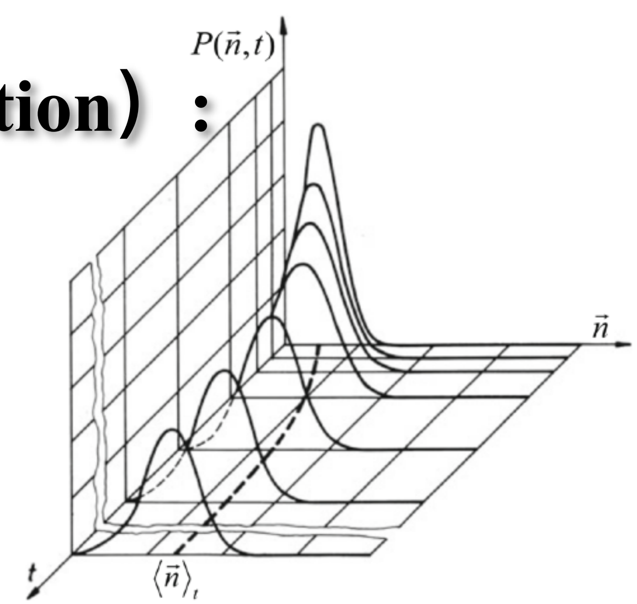
Nörenberg, Physics Letters B, Volume 53B, number 3

$$\frac{\partial P(x,t)}{\partial t} = \frac{\partial}{\partial x}[c_1(x,t)P(x,t)] + \frac{\partial^2}{\partial x^2}[c_2(x,t)P(x,t)] \ , \qquad (3)$$

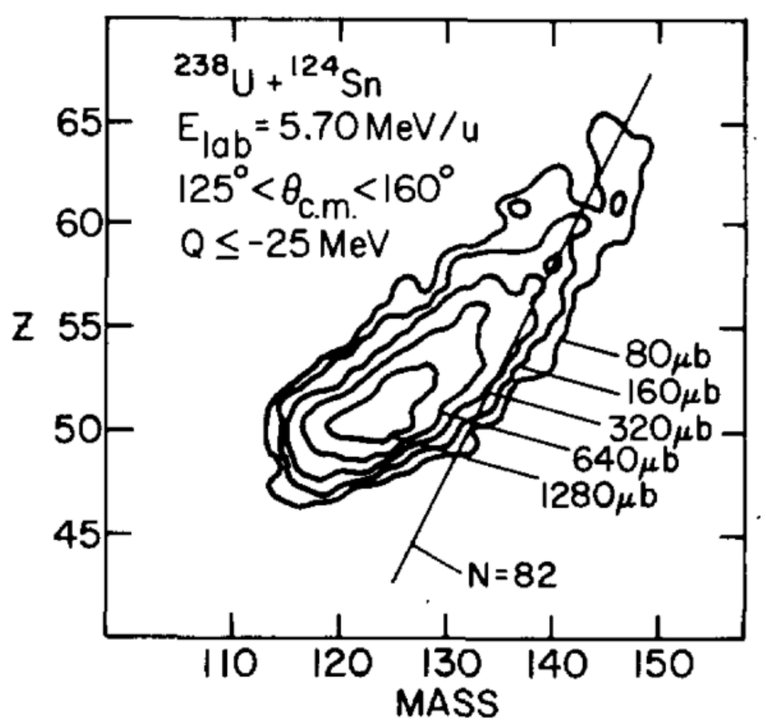
- **Drift:**

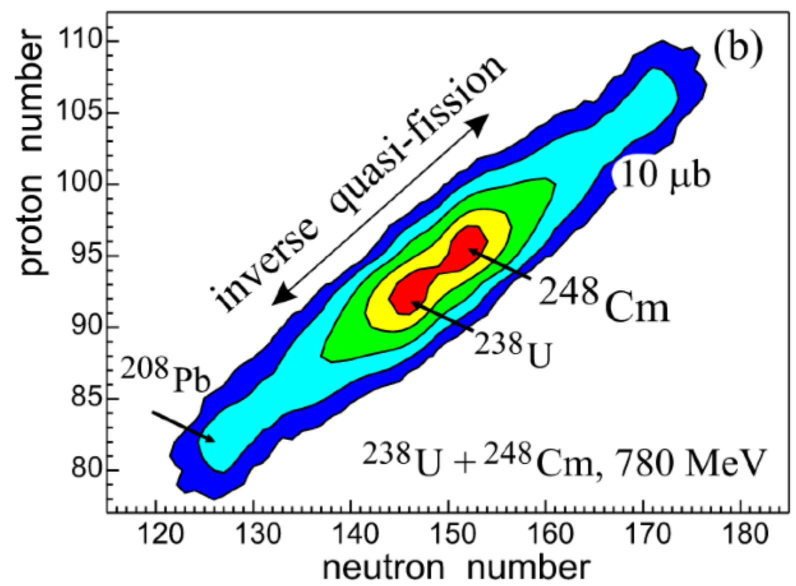


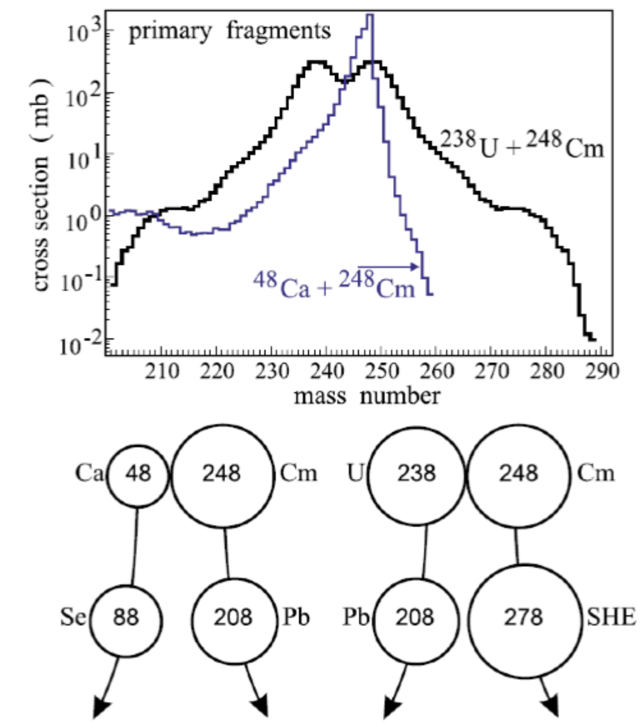
- **Diffusion （fluctuation）：**



Shell effect in Multinucleon transfer reactions







PHYSICS LETTERS; Volume 152B, number 3,4

Zagrebaev, W. Greiner / NPA 944 (2015) 257–307

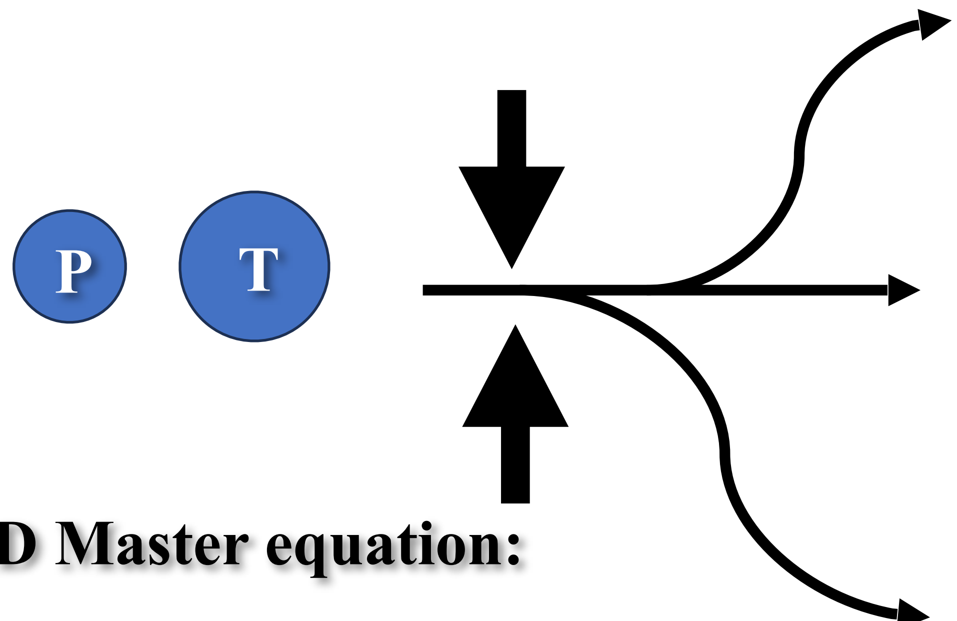
Zagrebaev, W. Greiner / NPA 944 (2015) 257–307

Shell effect plays an essential role for the nucleon exchange process

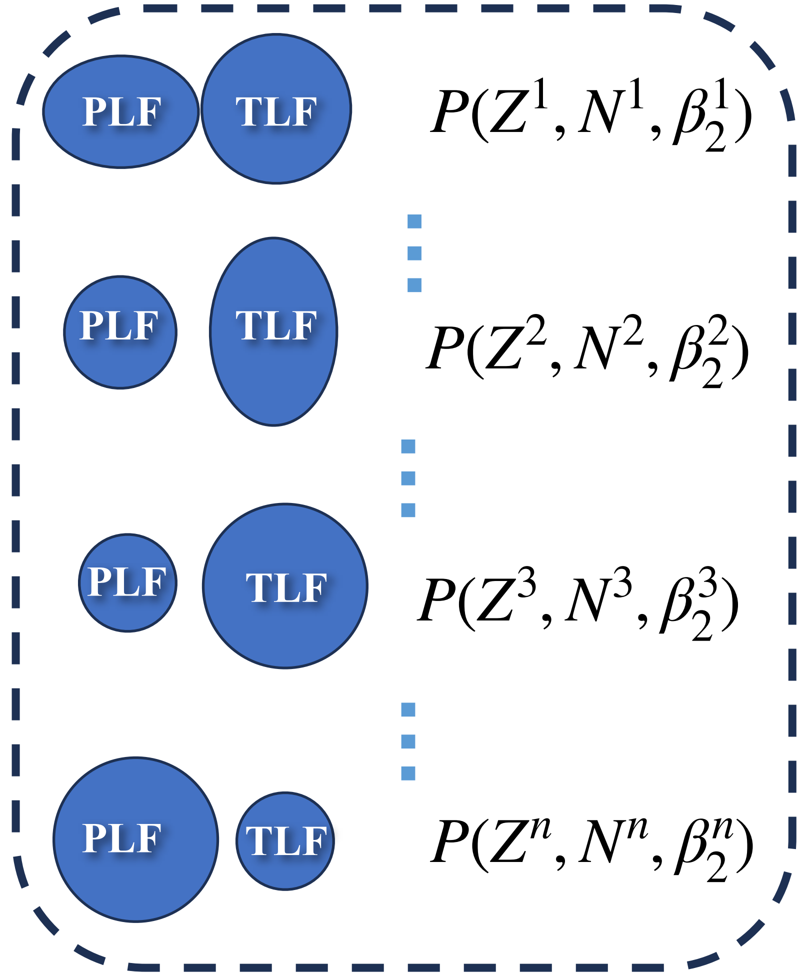
Model: DNS-sysu

Potential energy surface:

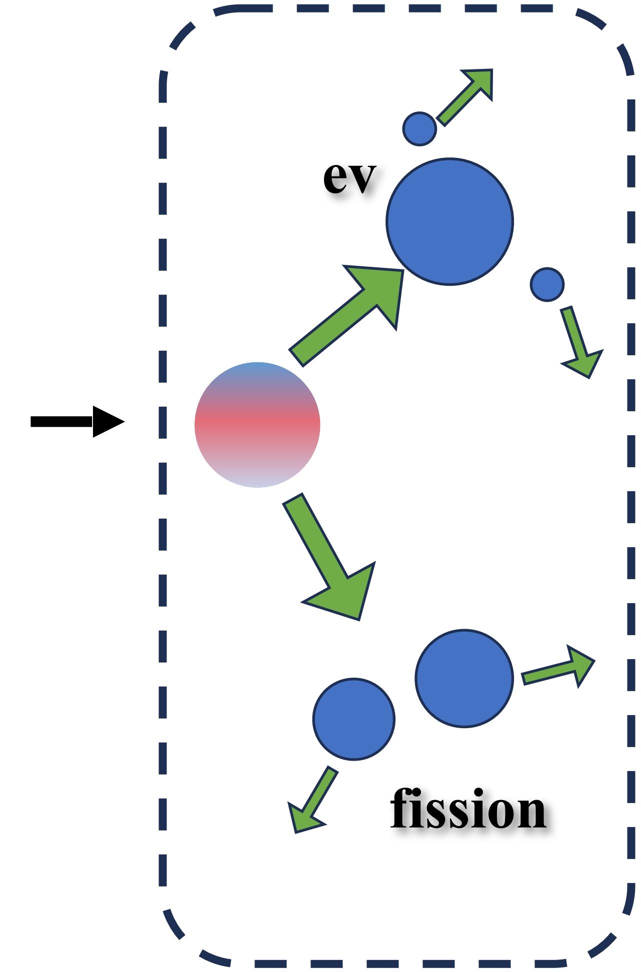
$$U(Z_1, N_1, \beta_2, J, r = R_{\text{cont}}) = \Delta(Z_1, N_1) + \Delta(Z_2, N_2) + V(Z_1, N_1, \beta_2, J, r = R_{\text{cont}}) + \tfrac{1}{2}C_1(\delta\beta_2^1)^2 + \tfrac{1}{2}C_2(\delta\beta_2^2)^2.$$



Configuration probability distribution :



Statistical model:



3D Master equation:

$$\begin{aligned}\frac{dP(Z_1, N_1, \beta_2, J, t)}{dt} &= \sum_{Z_1'} W_{Z_1,N_1,\beta_2;Z_1',N_1,\beta_2}(t)\big[d_{Z_1,N_1,\beta_2}P(Z_1', N_1, \beta_2, J, t) \\ &\quad - d_{Z_1',N_1,\beta_2}P(Z_1, N_1, \beta_2, J, t)\big] \\ &\quad + \sum_{N_1'} W_{Z_1,N_1,\beta_2;Z_1,N_1',\beta_2}(t)\big[d_{Z_1,N_1,\beta_2}P(Z_1, N_1', \beta_2, J, t) \\ &\quad - d_{Z_1,N_1',\beta_2}P(Z_1, N_1, \beta_2, J, t)\big] \\ &\quad + \sum_{\beta_2'} W_{Z_1,N_1,\beta_2;Z_1,N_1,\beta_2'}(t)\big[d_{Z_1,N_1,\beta_2}P(Z_1, N_1, \beta_2', J, t) \\ &\quad - d_{Z_1,N_1,\beta_2'}P(Z_1, N_1, \beta_2, J, t)\big].\end{aligned}$$

PLF: projectile like fragment

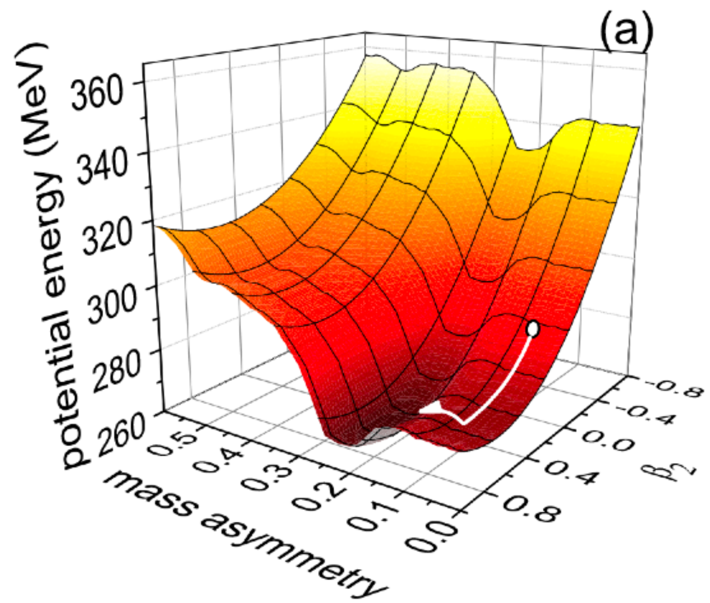
TLF: target like fragment

Model: DNS-sysu

PHYSICAL REVIEW C 97, 044614 (2018)

Potential energy surface:

$$U(Z_1,N_1,\beta_2,J,r=R_{\text{cont}}) = \Delta(Z_1,N_1)+\Delta(Z_2,N_2) + V(Z_1,N_1,\beta_2,J,r=R_{\text{cont}}) + \tfrac{1}{2}C_1(\delta\beta_2^1)^2 + \tfrac{1}{2}C_2(\delta\beta_2^2)^2.$$



Liquid drop parameters:

$$\Delta(Z_i,N_i) = Z_i\Delta(^1H) + N_i\Delta(n) - a_v(1-\kappa I^2)A_i + a_s(1-\kappa I^2)A_i^{2/3} + a_c Z_i^2 A_i^{-1/3} - c_4 Z_i^2 A_i^{-1} - E_{\text{pair}}(Z_i,N_i) + E_{\text{sh}}(Z_i,N_i), \quad (5)$$

α_v	α_s	α_c	k	c_4
15.677 MeV	18.56 MeV	0.717 MeV	1.79	1.211 MeV

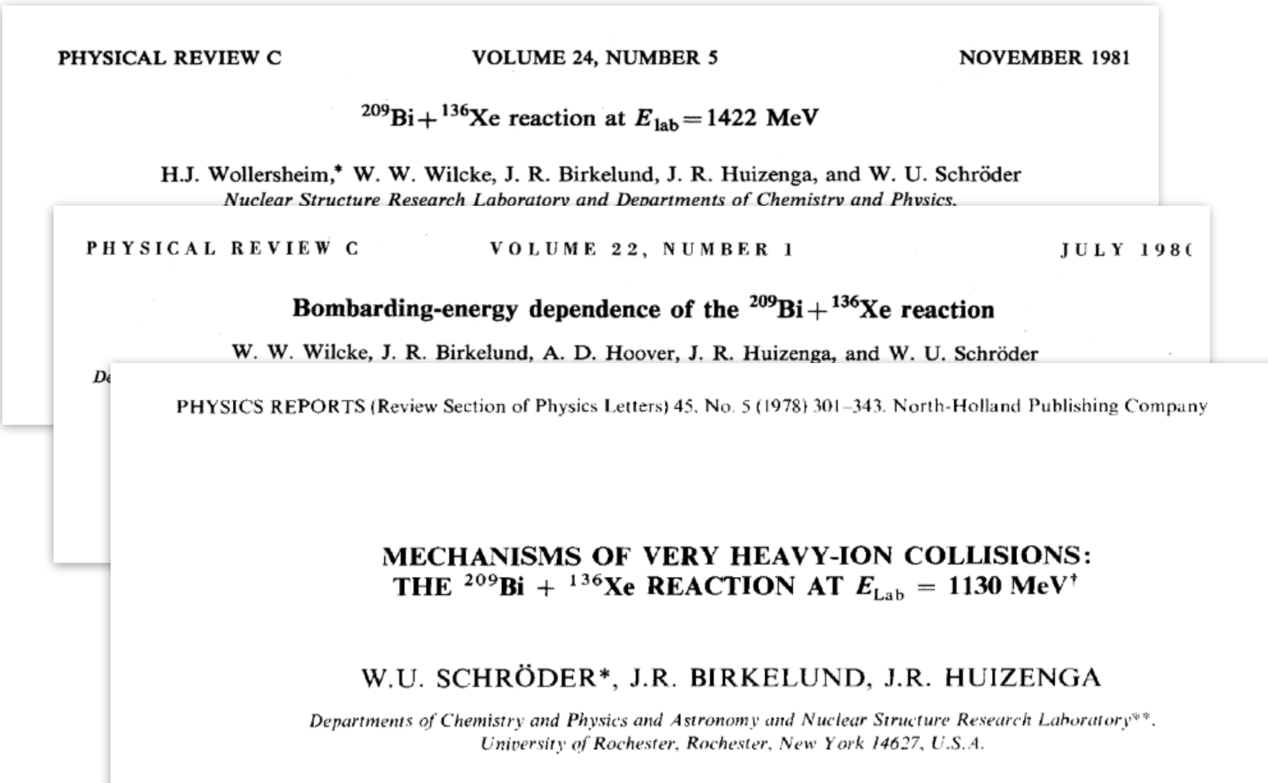
Temperature dependence:

V. M. Strutinsky, Nucl. Phys. A 95, 420 (1967).

$$E_{\text{sh}}(Z_i,N_i) = E_{\text{sh}}^0(Z_i,N_i)e^{-E^*/E_d}.$$

$$E^* = E_{diss} \times A_i/A_{tot} \qquad E_d = \mathbf{5.48}\ A_i^{1/3}/(1+1.3A_i^{1/3})$$

Experiment：136Xe+209Bi



PHYSICAL REVIEW C VOLUME 24, NUMBER 5 NOVEMBER 1981

^{209}Bi+^{136}Xe reaction at $E_{\rm lab}=1422$ MeV

H.J. Wollersheim,* W. W. Wilcke, J. R. Birkelund, J. R. Huizenga, and W. U. Schröder

Nuclear Structure Research Laboratory and Departments of Chemistry and Physics,

PHYSICAL REVIEW C VOLUME 22, NUMBER 1 JULY 198(

Bombarding-energy dependence of the ^{209}Bi+^{136}Xe reaction

W. W. Wilcke, J. R. Birkelund, A. D. Hoover, J. R. Huizenga, and W. U. Schröder

PHYSICS REPORTS (Review Section of Physics Letters) 45, No. 5 (1978) 301–343. North-Holland Publishing Company

MECHANISMS OF VERY HEAVY-ION COLLISIONS:
THE ^{209}Bi + ^{136}Xe REACTION AT $E_{\rm Lab}$ = 1130 MeV†

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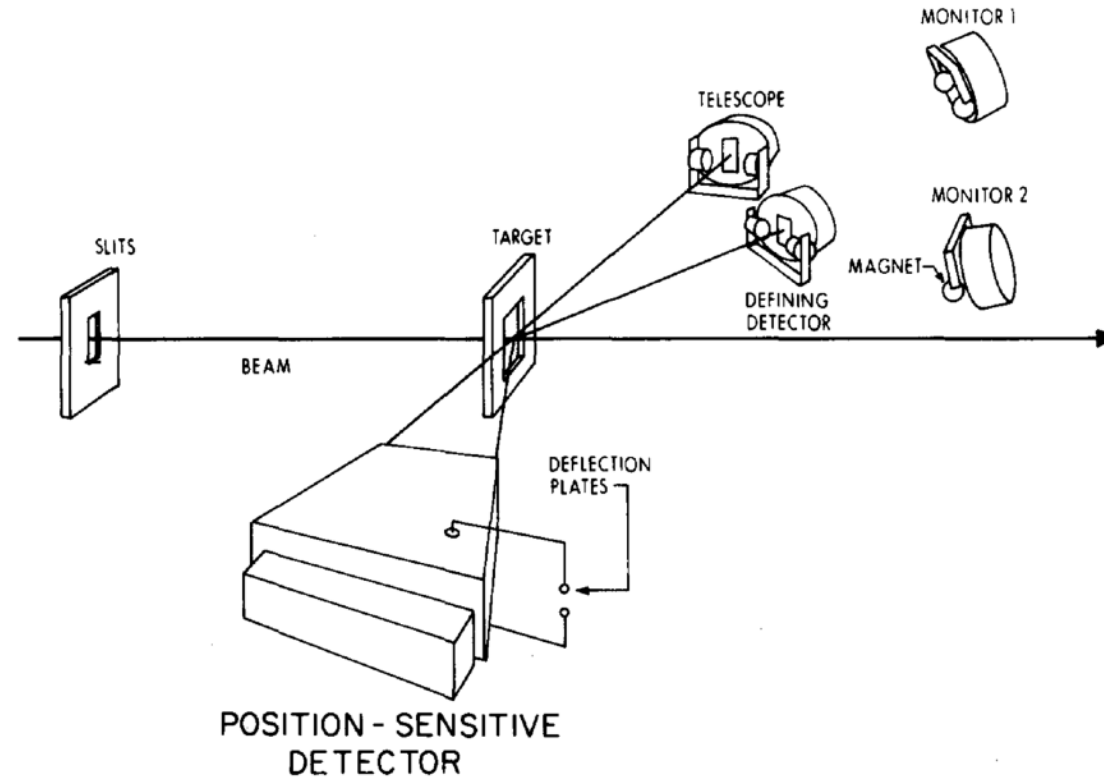
Departments of Chemistry and Physics and Astronomy and Nuclear Structure Research Laboratory**,
University of Rochester, Rochester, New York 14627, U.S.A.



W. W. Wilcke. et al. , Phys.Rev.C22, 128 (1980).
W. U. Schröder, et al. Phys. Rep. 45, 301 (1978).

Abundant experimental data：

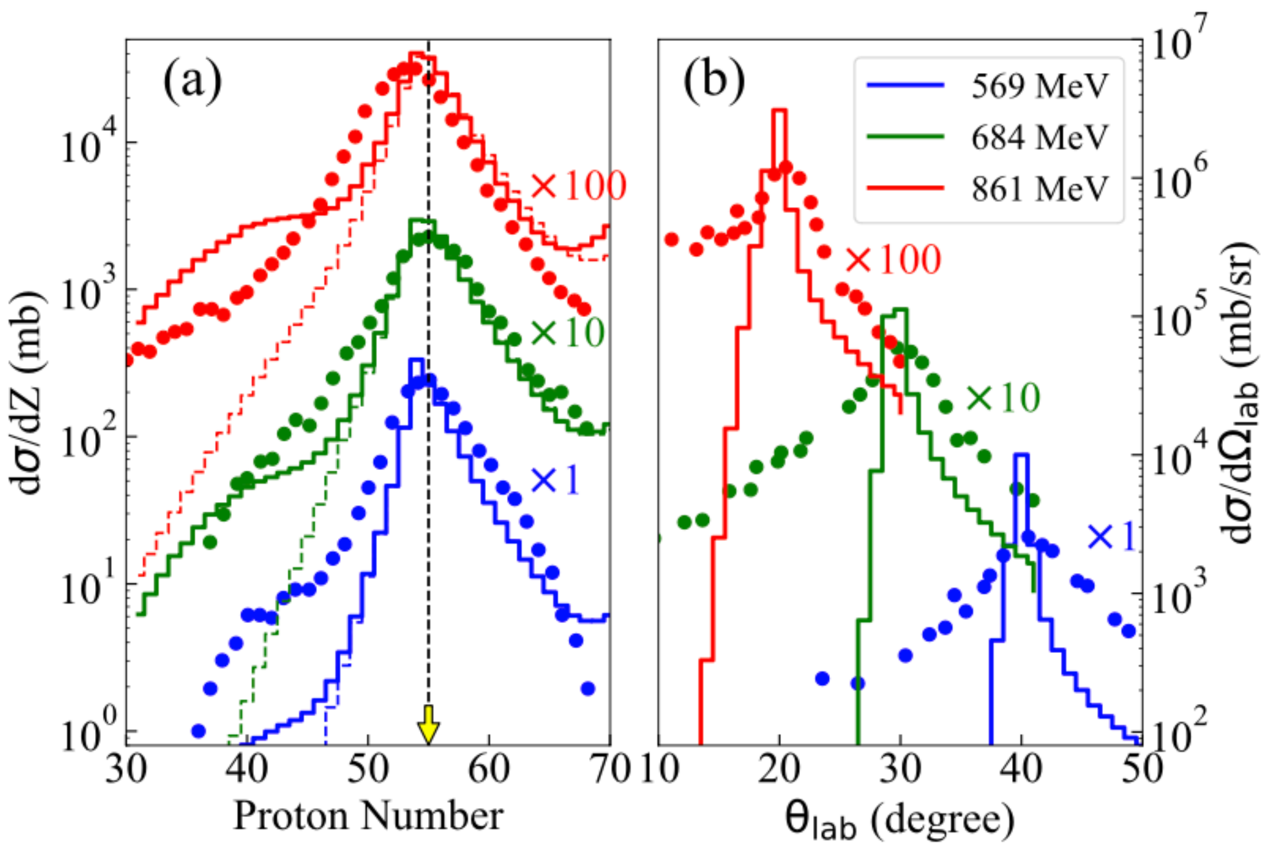
charge distribution, angular distribution,

mean value, variance, energy loss

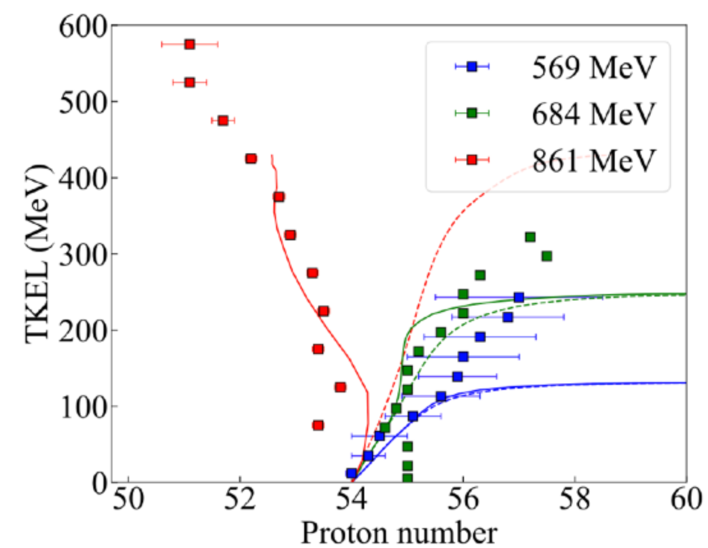
An ideal MNT reaction
to study the transport behavior of drift and fluctuation

Comparison: Cal & Exp

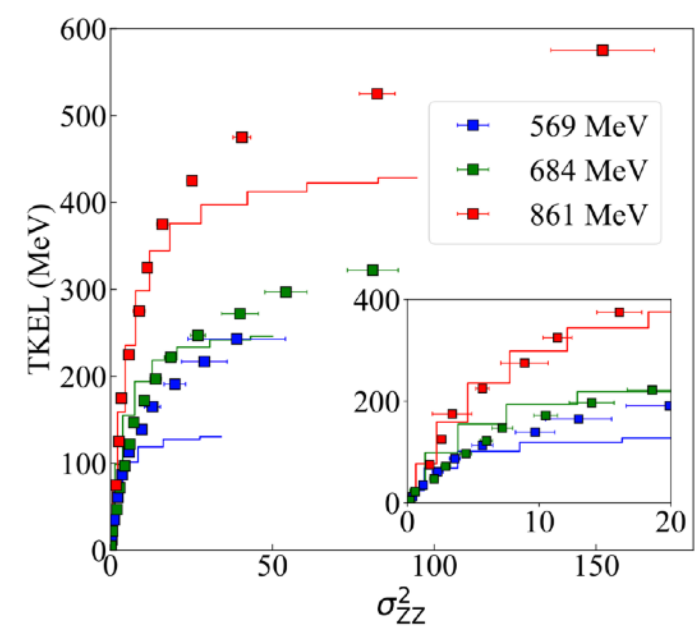
Cross section & Angular Distribution:



Charge mean value:



Charge variance value:



➢ Reasonable description of various observations

➢ TKEL is insufficient, the model needs improvement

Exp data from W. W. Wilcke. et al. , Phys.Rev.C22, 128 (1980).

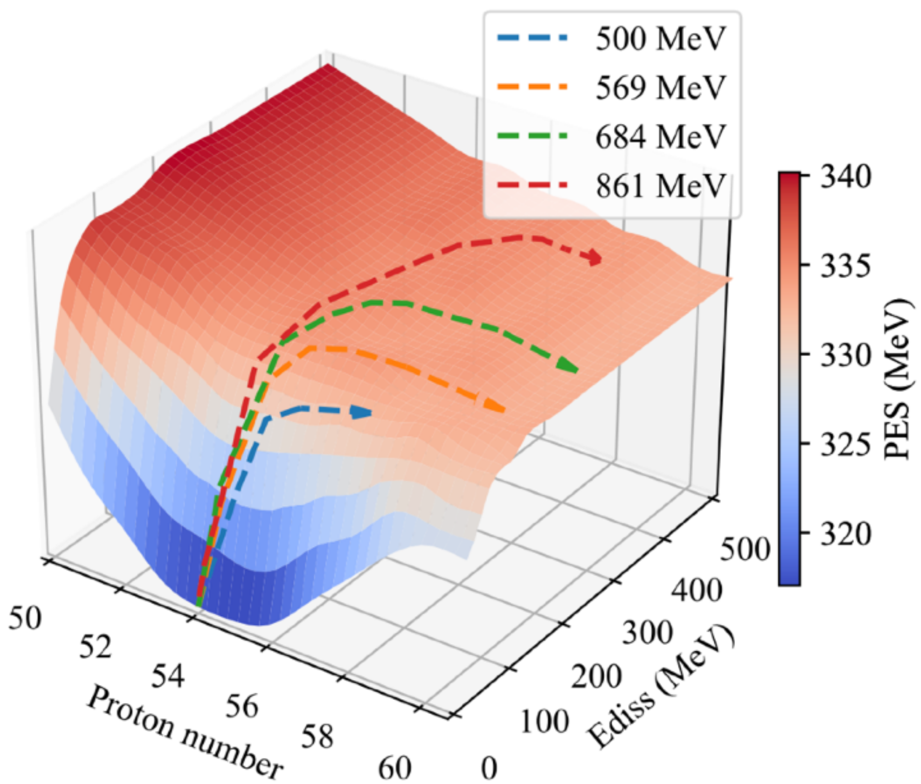
W. U. Schröder, et al. Phys. Rep. 45, 301 (1978).

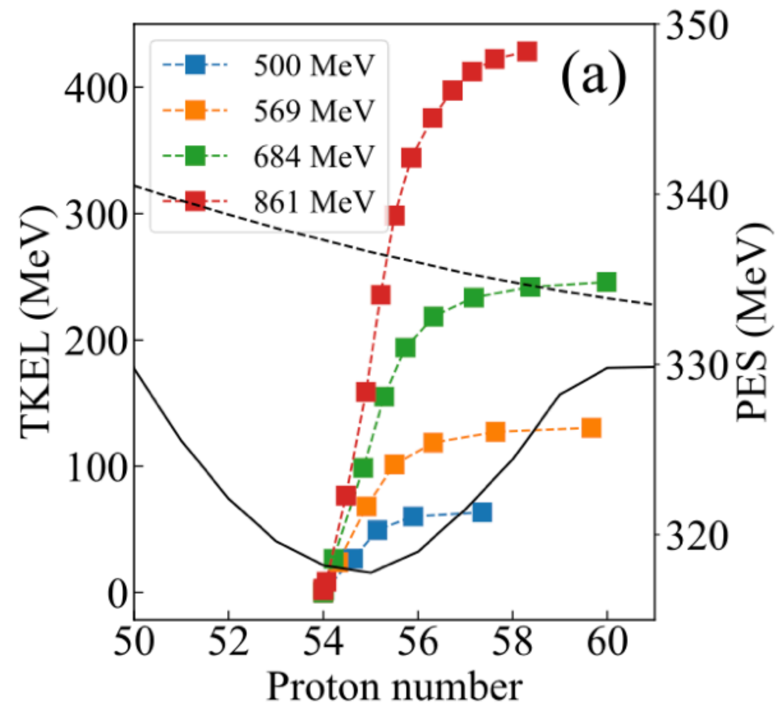
Drift in the MNT reaction

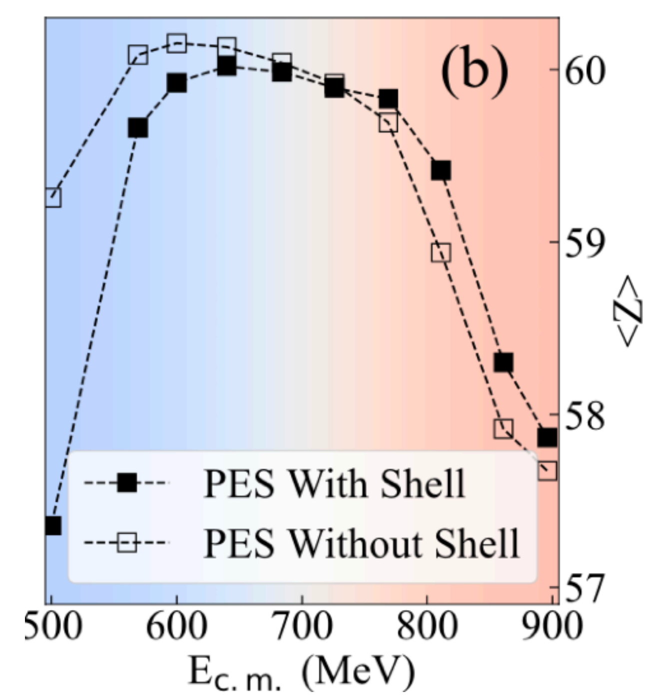
Drift path & Potential energy surface:

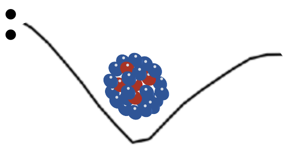
$$\bar{Z}_1(t) = \langle Z_1(t) \rangle = \sum_{Z_1} \sum_{N_1} \sum_{\beta_2} Z_1 \times P(Z_1, N_1, \beta_2, t)$$

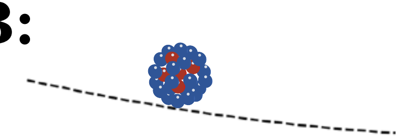
$$\bar{N}_1(t) = \langle N_1(t) \rangle = \sum_{Z_1} \sum_{N_1} \sum_{\beta_2} N_1 \times P(Z_1, N_1, \beta_2, t).$$

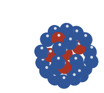






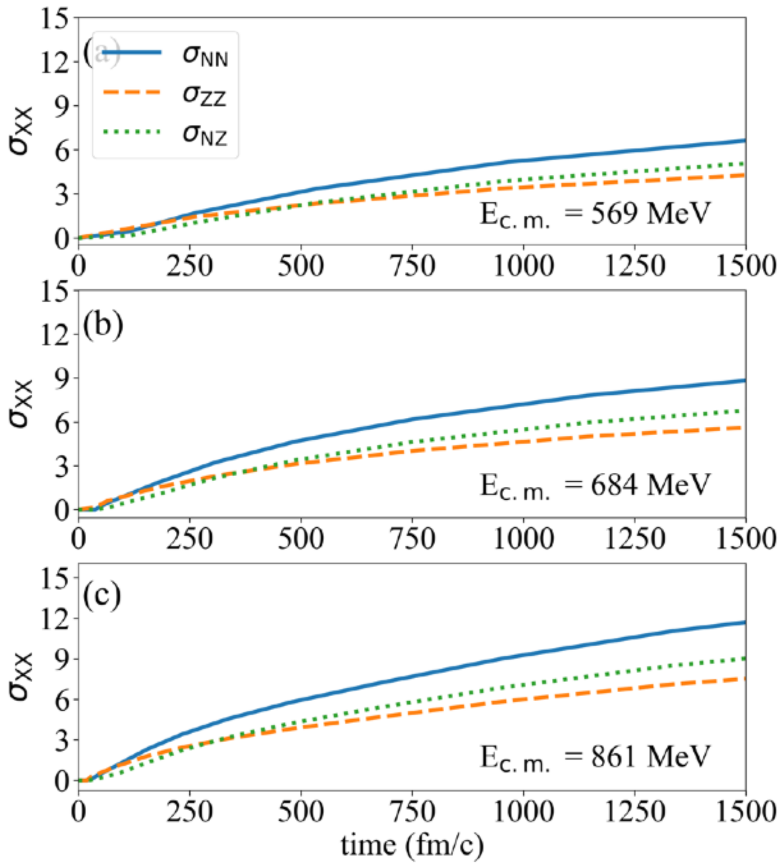
A: 

B: 

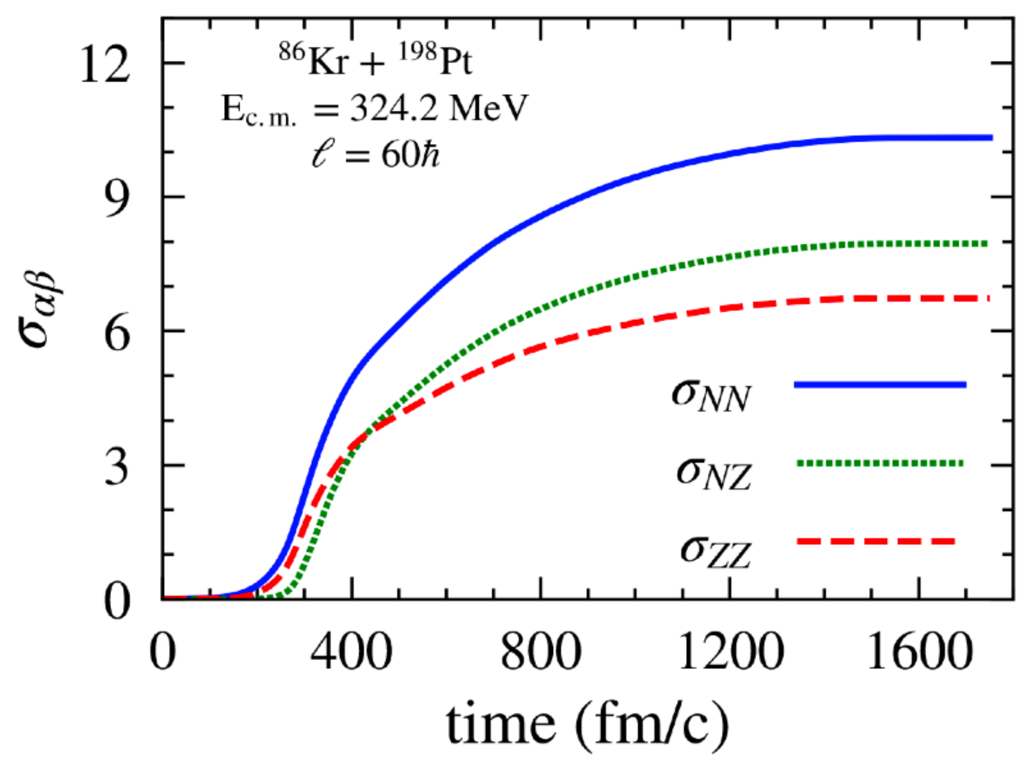
C: 

Fluctuation in the MNT reaction

- **Variance:**



M. Arik, PHYSICAL REVIEW C 108, 064604 (2023).

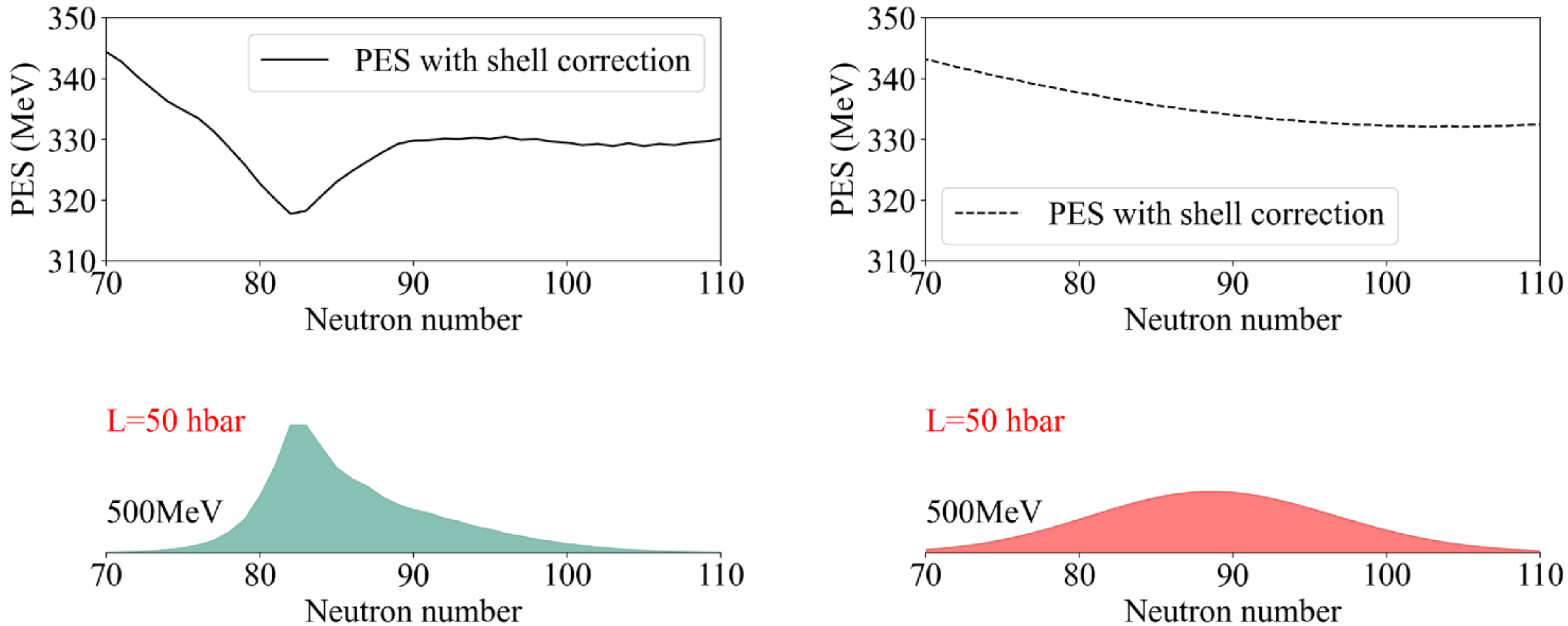


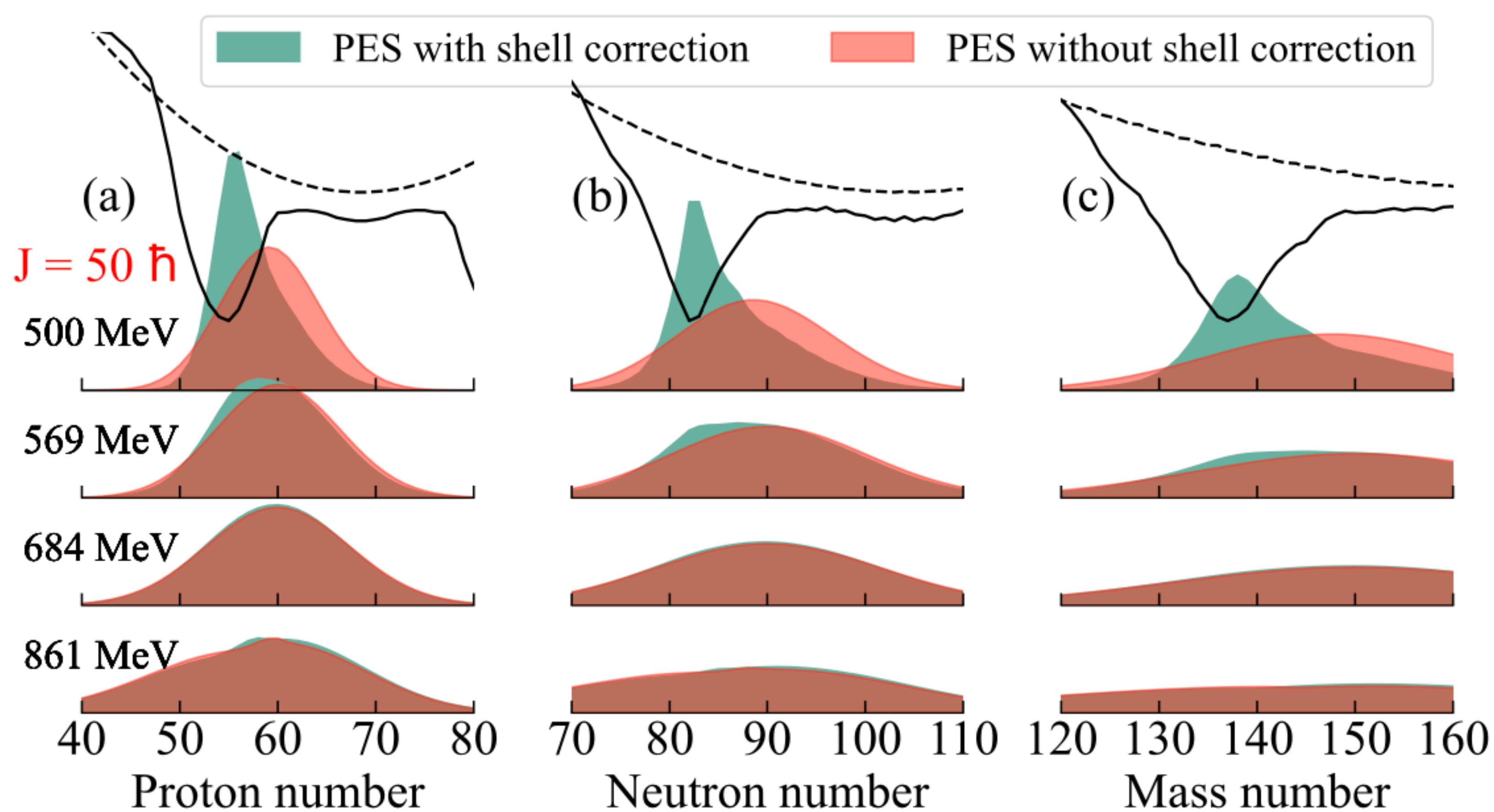
$$\sigma^2_{NN}(t) = \sum_{Z_1}\sum_{N_1}\sum_{\beta_2}(N_1 - \bar{N}_1(t))^2 \times P(Z_1, N_1, \beta_2, t)$$

$$\sigma^2_{ZZ}(t) = \sum_{Z_1}\sum_{N_1}\sum_{\beta_2}(Z_1 - \bar{Z}_1(t))^2 \times P(Z_1, N_1, \beta_2, t)$$

$$\sigma^2_{NZ}(t) = \sum_{Z_1}\sum_{N_1}\sum_{\beta_2}(N_1 - \bar{N}_1(t))(Z_1 - \bar{Z}_1(t)) \times P(Z_1, N_1, \beta_2, t)$$

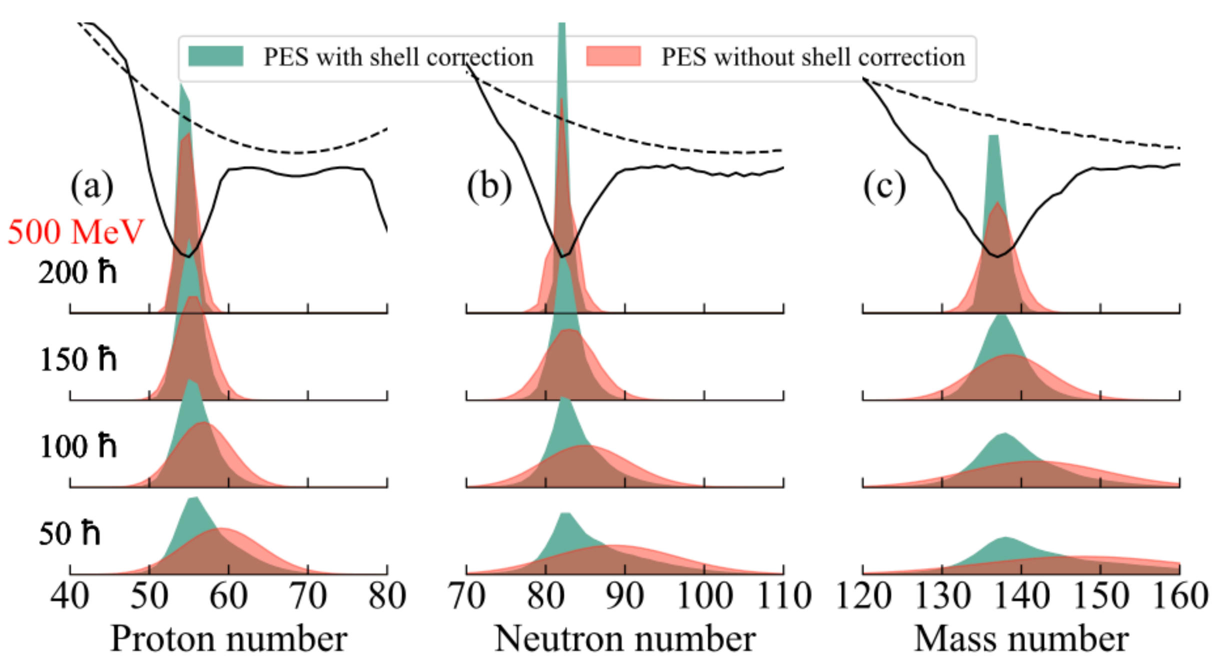
Shell effect on the production distribution



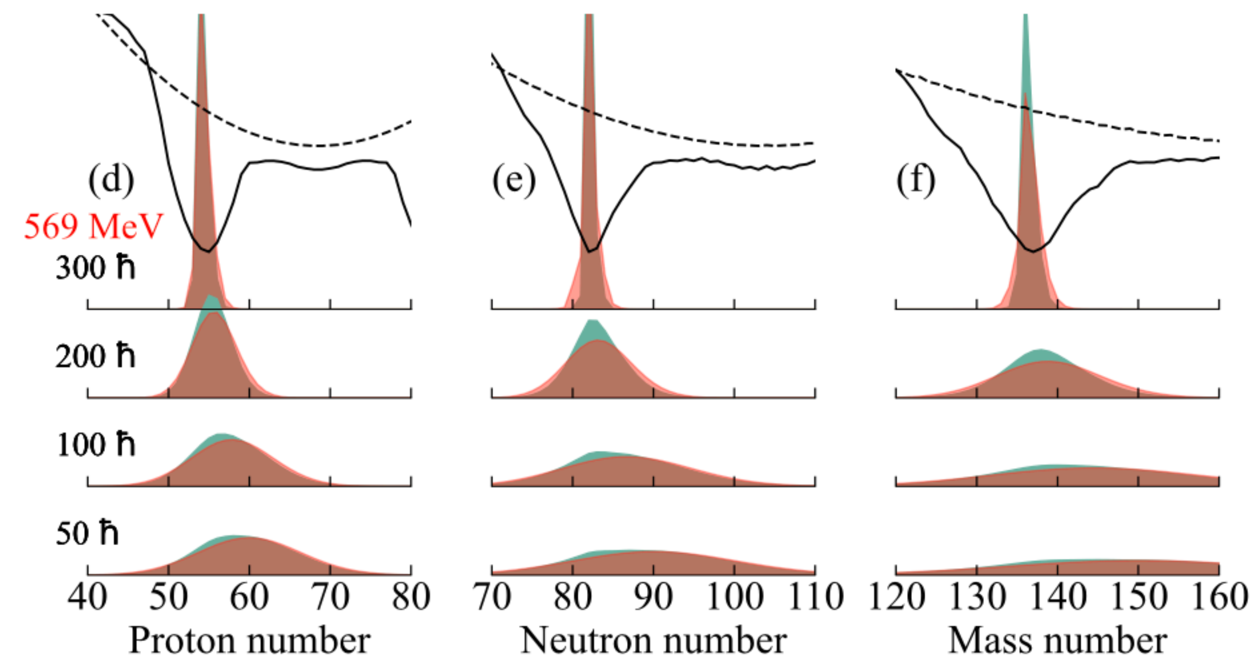


Shell effect on the production distribution

- $E_{c.m.} = 500$ MeV:



- $E_{c.m.} = 569$ MeV:



Summary

- With increasing system temperature, the shell effect gradually diminishes, leading to increased nucleon transfer as the shell attraction fades.

- However, at high incident energies, the constraining effect of the potential energy surface (PES) on system evolution weakens, causing a reversal in the evolution direction.

- Furthermore, the consideration of shell corrections in the PES significantly impact both average values and variances of fragments. However, this influence diminished in high-energy conditions.

Thank you for your attention!

Model: DNS-sysu

Hamiltonian:

$$H(t) = H_0(t) + V(t),$$

$$H_0(t) = \sum_K \sum_{\nu_K} \varepsilon_{\nu_K}(t) a^+_{\nu_K}(t) a_{\nu_K}(t),$$

$$V(t) = \sum_{K,K'} \sum_{\alpha_K,\beta'_K} u_{\alpha_K,\beta'_K}(t) a^+_{\alpha_K}(t) a_{\beta'_K}(t) = \sum_{K,K'} V_{K,K'}(t),$$

Space nucleon:

$$\Delta\varepsilon_K = \sqrt{\frac{4\varepsilon^*_K}{g_K}}, \quad \varepsilon^*_K = \varepsilon^* \frac{A_K}{A}, \quad g_K = \frac{A_K}{12}.$$

Transition probability :

$$W_{\xi,\xi'}(t) = \frac{\tau_{\mathrm{mem}}(\xi,\xi')}{\hbar^2 d_\xi d_{\xi'}} \sum_{ii'} |\langle \xi', i' | V | \xi, i \rangle|^2$$

The strength parameters

$$U_{kk'} = \frac{g_1^{\frac{1}{3}} g_2^{\frac{1}{3}}}{g_1^{\frac{1}{3}} + g_2^{\frac{1}{3}}} \frac{1}{g_k^{\frac{1}{3}} g_{k'}^{\frac{1}{3}}} 2\gamma_{kk'}$$

The single-particle matrix elements :

$$u_{\alpha_K,\beta_{K'}}(t) = U_{K,K'}(t)\left\{ \exp\left[-\frac{1}{2}\left(\frac{\varepsilon_{\alpha_K}(t) - \varepsilon_{\beta_{K'}}(t)}{\Delta_{K,K'}(t)} \right)^2 \right] - \delta_{\alpha_K,\beta_{K'}} \right\}$$