

多核子转移反应研讨会

多核子转移反应角分布的研究



湖南·长沙

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导师：祝龙

Sino-French Institute of Nuclear Engineering and Technology

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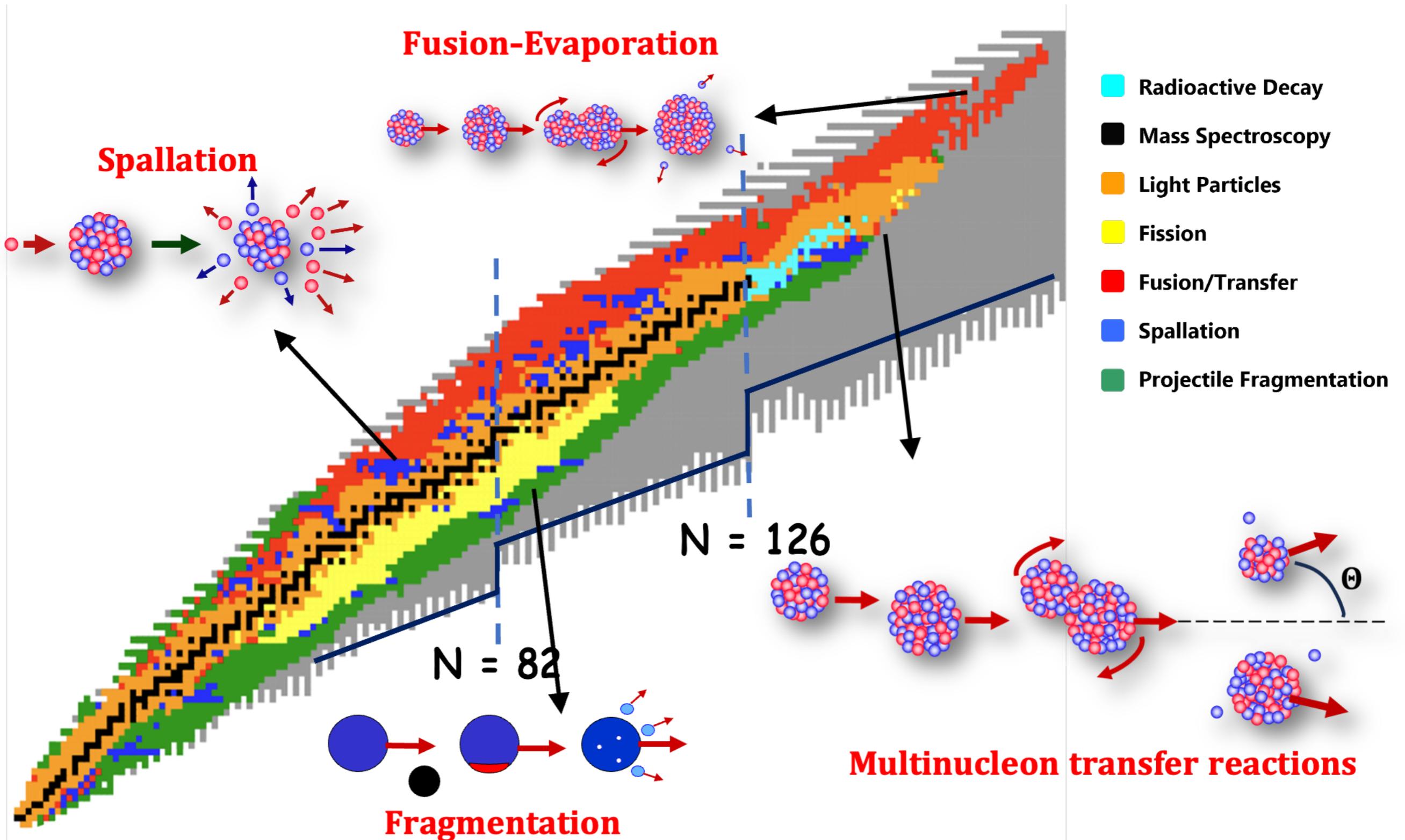
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■ 多核子转移反应的角分布

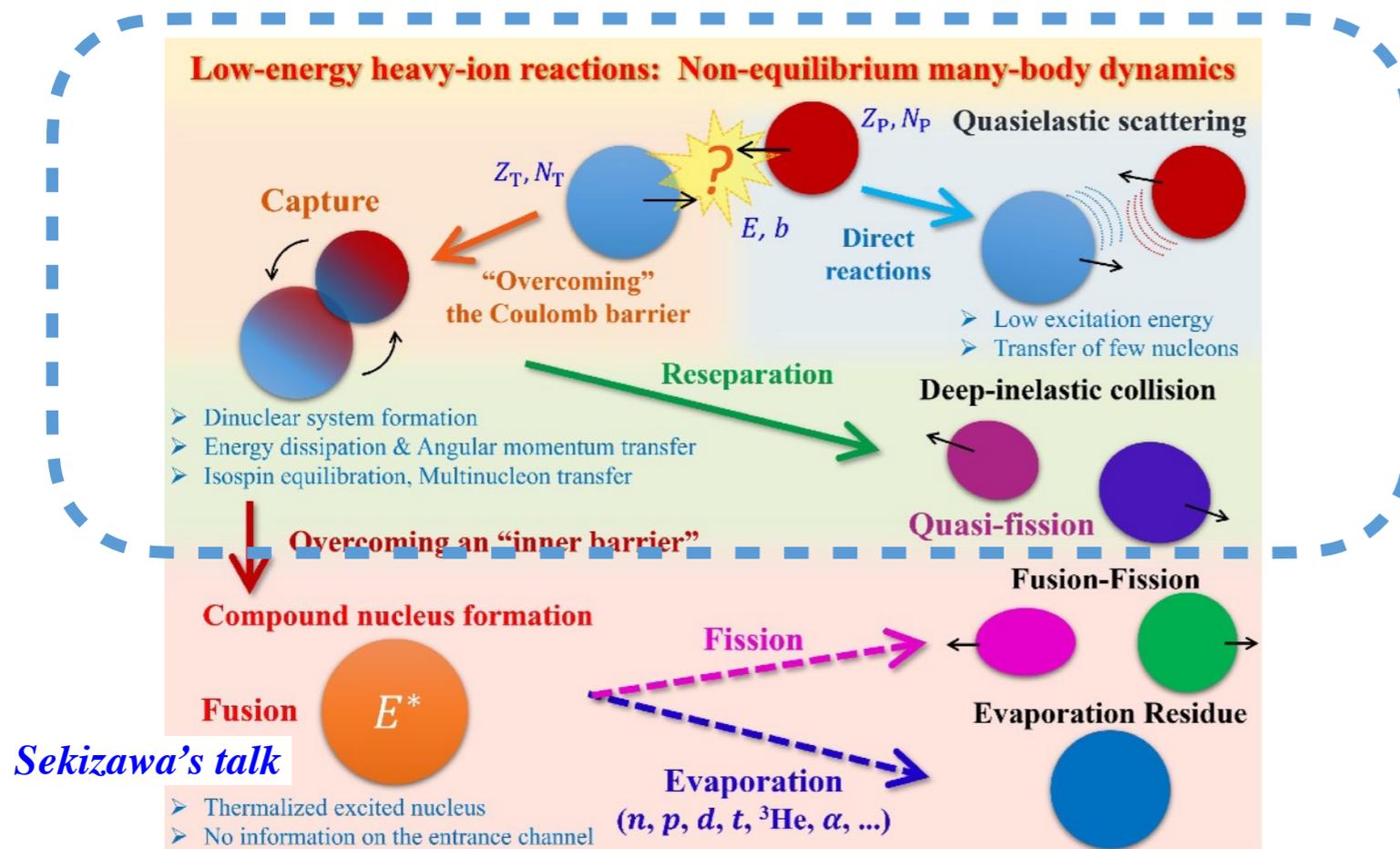
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多核子转移反应-新核素合成



多核子转移反应 · 特征

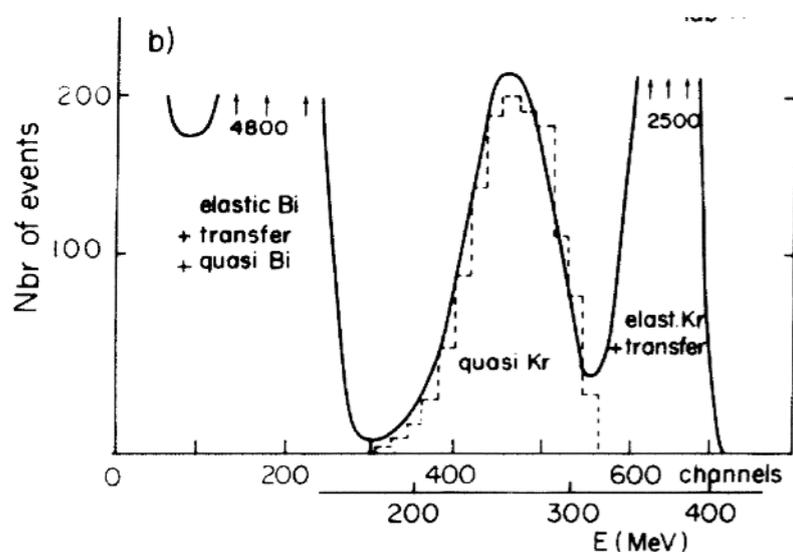


■ 大的能量耗散

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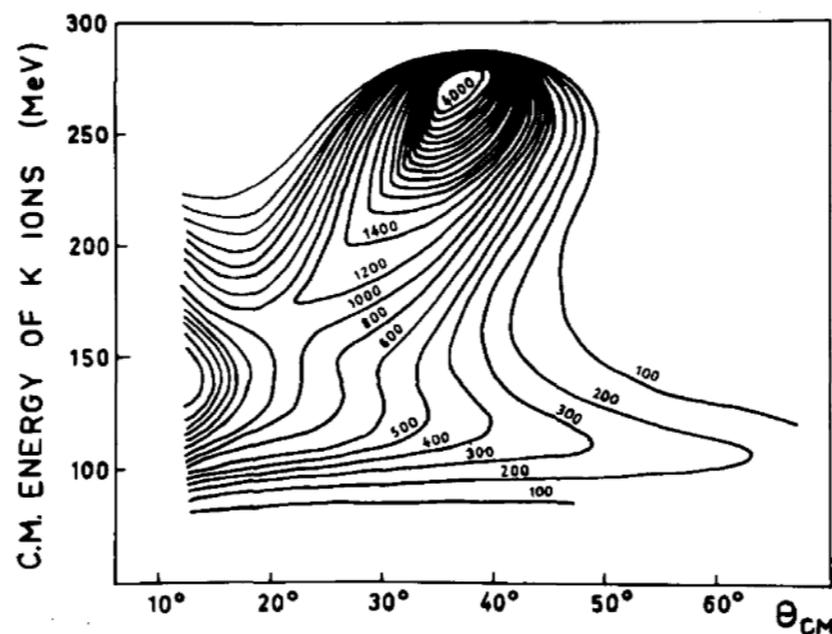
■ 众多的反应道

${}^{209}\text{Bi} + {}^{84}\text{Kr}$ at energy 525 MeV



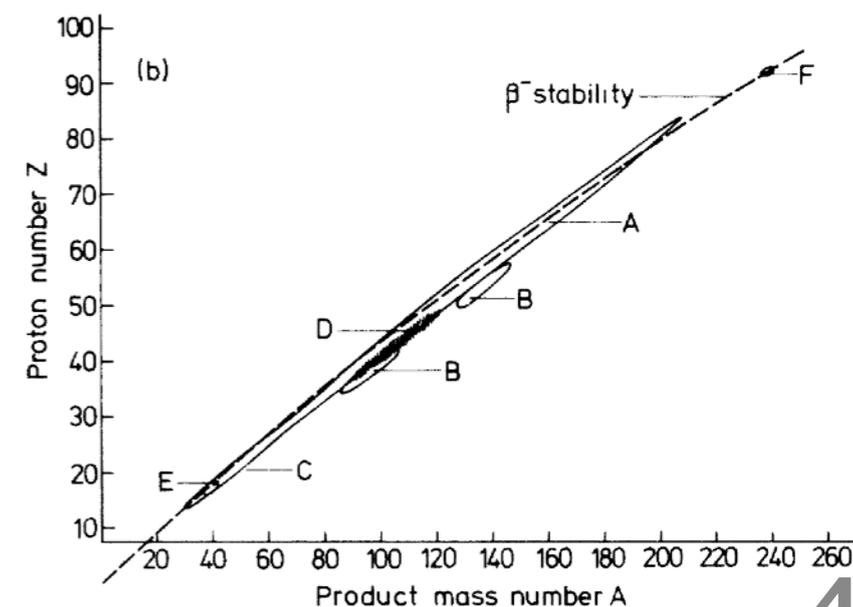
Phys. Rev. Lett. 32. 738 (1974)

${}^{232}\text{Th}({}^{40}\text{Ar}, \text{K})$ at energy 388 MeV



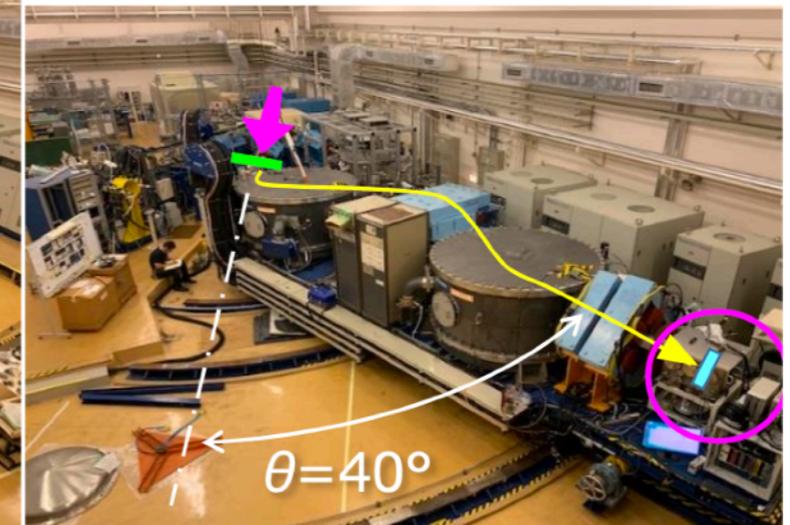
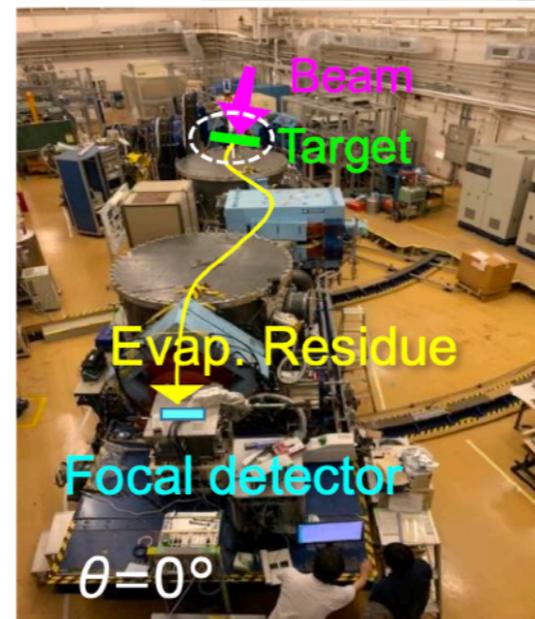
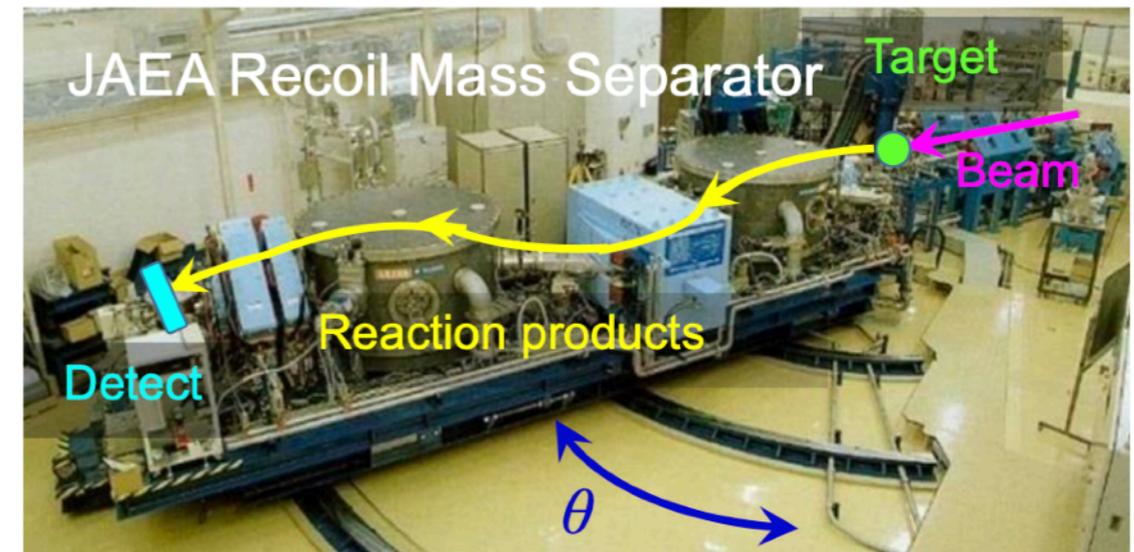
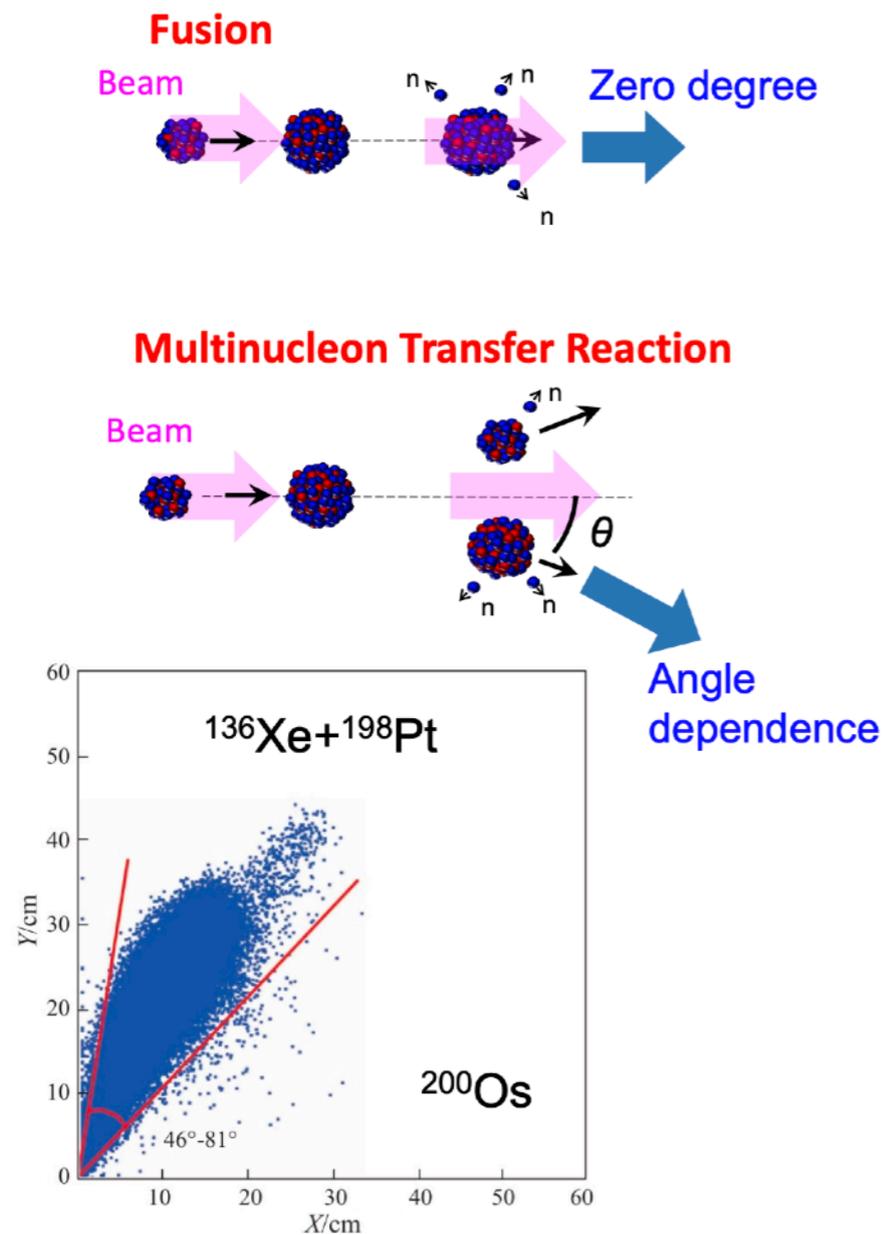
Phys. Lett. 47B (1973) 484

${}^{238}\text{U} + {}^{40}\text{Ar}$ at energy 288 MeV



Phys. RevC. 13. 2347 (1976)

多核子转移反应-角分布



Wenxue Huang et al. Nucl. Phys. Rev. (2017)

H. Ikezoe et al., Nucl. Instrum. Meth. A 376, 420 (1996).

MNT反应产物的出射在实验室系并不是 0° 附近的前冲方向，而是覆盖了一个大范围的圆锥角

这给收集和分离我们感兴趣的多核子转移反应产物带来了很大的困难，需要理论支持。

理论计算可以对实验探测方位角的选取提供理论指导

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■ 多核子转移反应的角分布

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- 基于半经验公式研究多核子转移角分布

■ 总结与展望

主方程

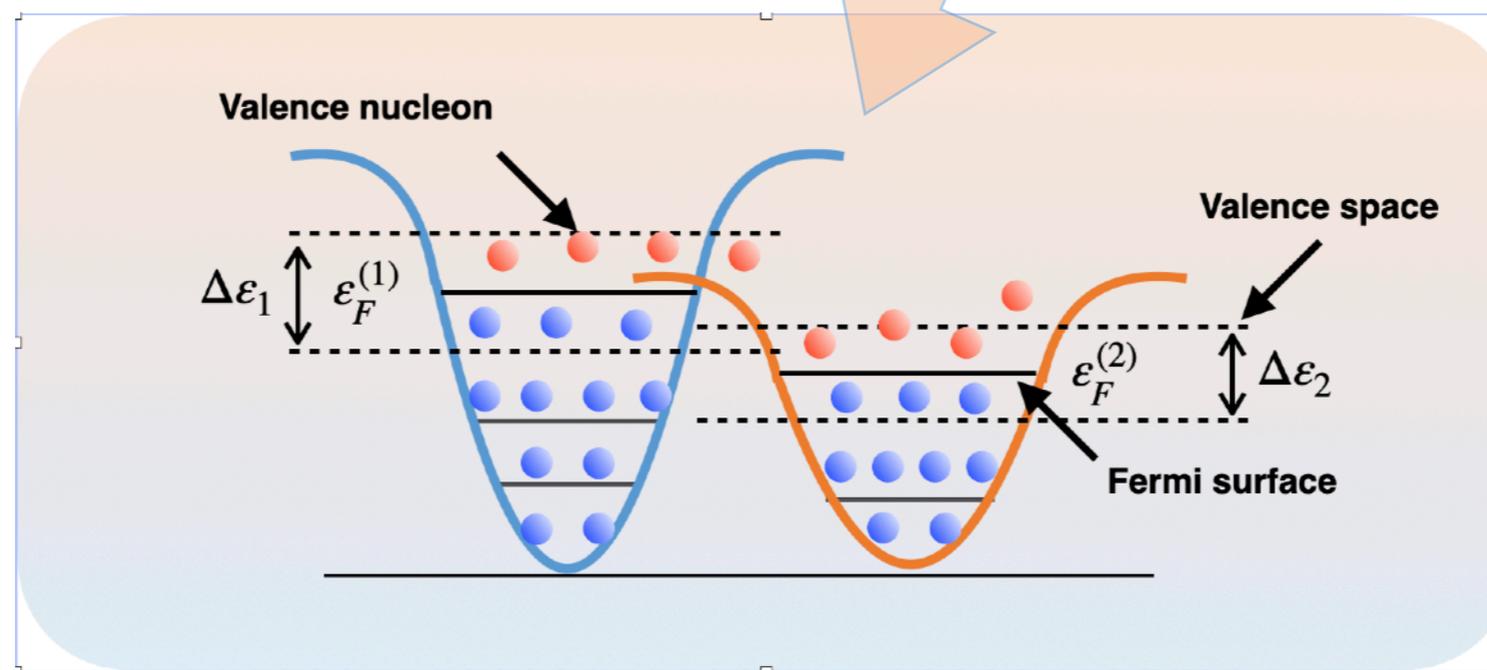
● 主方程:

$$dP_N(t)/dt = \sum_M w_{MN}(t) \{NP_M(t) - MP_N(t)\}$$

- N : the number of channels.
- $P_N(t)$: the sum over all occupation probabilities of the channels in the subset $\{N\}$.
- $w_{MN}(t)$: the mean transition probability from a channel $n \in \{N\}$ to a channel $m \in \{M\}$.

仅考虑单核子转移:

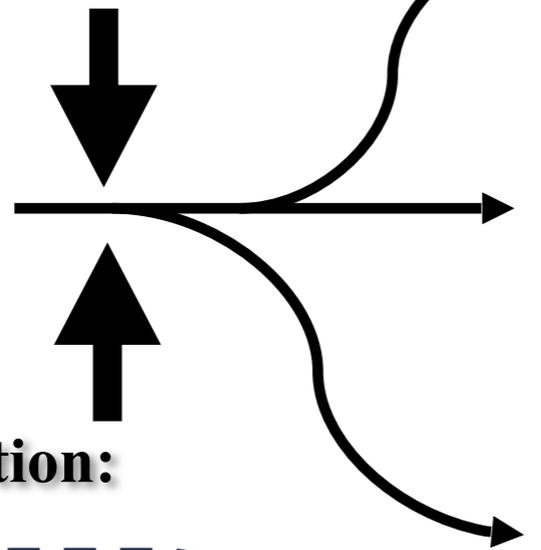
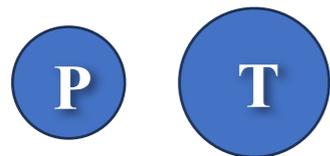
$$\begin{pmatrix} P_1(t + \Delta t) \\ P_2(t + \Delta t) \\ P_3(t + \Delta t) \\ \vdots \\ P_n(t + \Delta t) \end{pmatrix} = \begin{pmatrix} \begin{pmatrix} 1 & 0 & 0 & \dots & 0 \\ 0 & 1 & 0 & \dots & 0 \\ 0 & 0 & 1 & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & 1 \end{pmatrix} + \Delta t \begin{pmatrix} -W_{11} & W_{12} & 0 & \dots & 0 \\ W_{21} & -W_{22} & W_{23} & \dots & 0 \\ 0 & W_{32} & -W_{33} & \dots & 0 \\ \vdots & \vdots & \vdots & \ddots & \vdots \\ 0 & 0 & 0 & \dots & -W_n \end{pmatrix} \end{pmatrix} \begin{pmatrix} P_1(t) \\ P_2(t) \\ P_3(t) \\ \vdots \\ P_n(t) \end{pmatrix}$$



双核模型

□ Potential energy surface:

$$U(Z_1, N_1, \beta_2, J, r = R_{\text{cont}}) = \Delta(Z_1, N_1) + \Delta(Z_2, N_2) + V(Z_1, N_1, \beta_2, J, r = R_{\text{cont}}) + \frac{1}{2}C_1(\delta\beta_1^1)^2 + \frac{1}{2}C_2(\delta\beta_2^2)^2.$$

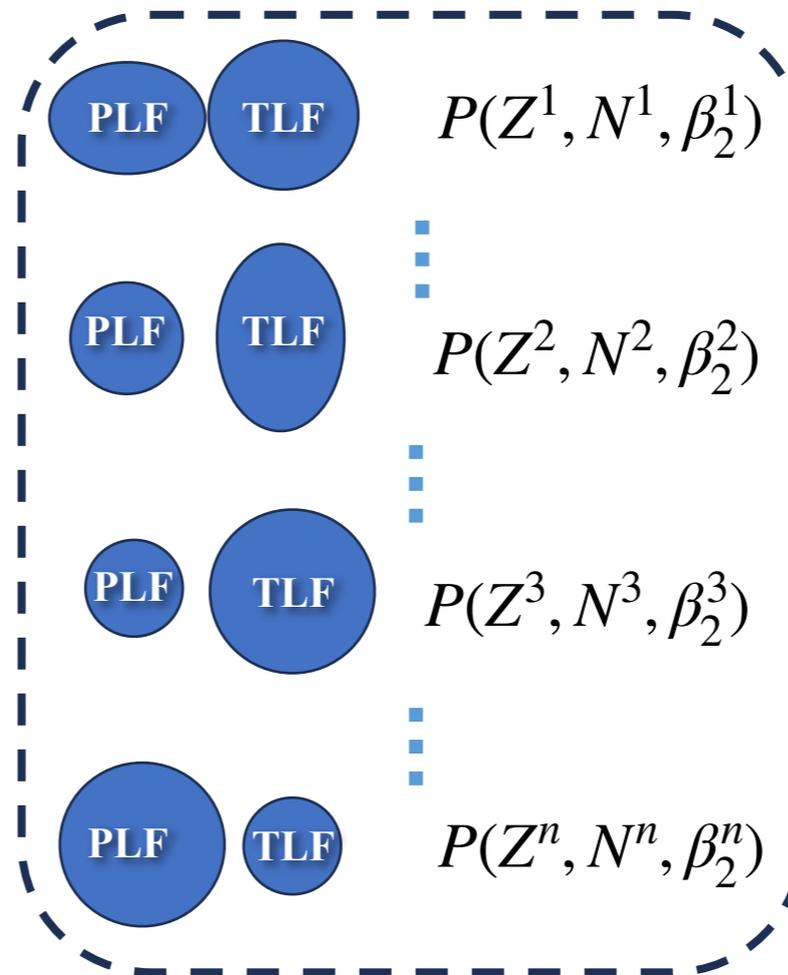


□ 3D Master equation:

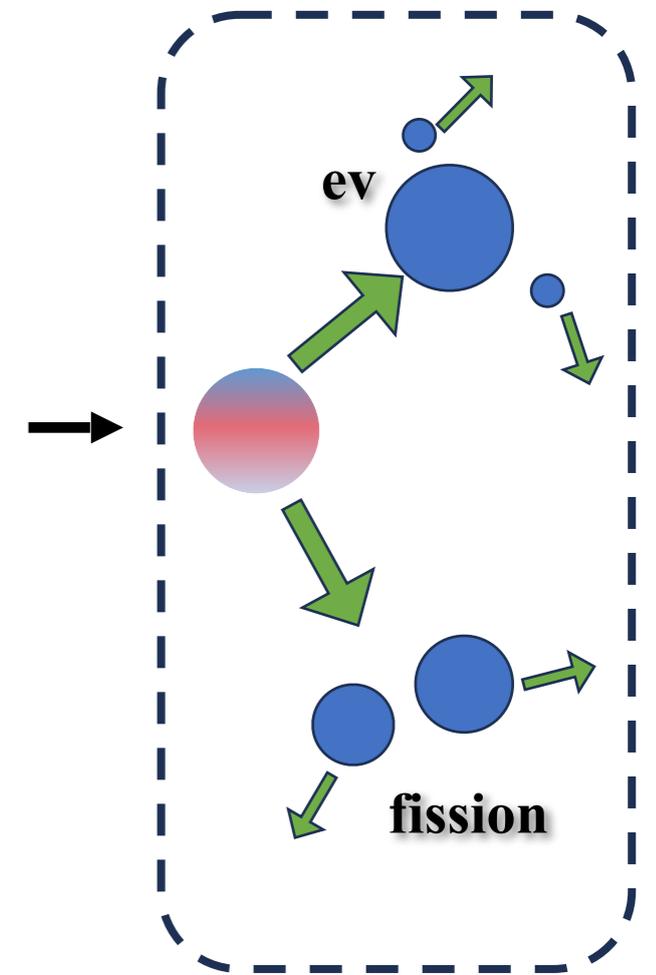
$$\begin{aligned} & \frac{dP(Z_1, N_1, \beta_2, J, t)}{dt} \\ &= \sum_{Z_1'} W_{Z_1, N_1, \beta_2; Z_1', N_1, \beta_2}(t) [d_{Z_1, N_1, \beta_2} P(Z_1', N_1, \beta_2, J, t) \\ & \quad - d_{Z_1', N_1, \beta_2} P(Z_1, N_1, \beta_2, J, t)] \\ &+ \sum_{N_1'} W_{Z_1, N_1, \beta_2; Z_1, N_1', \beta_2}(t) [d_{Z_1, N_1, \beta_2} P(Z_1, N_1', \beta_2, J, t) \\ & \quad - d_{Z_1, N_1', \beta_2} P(Z_1, N_1, \beta_2, J, t)] \\ &+ \sum_{\beta_2'} W_{Z_1, N_1, \beta_2; Z_1, N_1, \beta_2'}(t) [d_{Z_1, N_1, \beta_2} P(Z_1, N_1, \beta_2', J, t) \\ & \quad - d_{Z_1, N_1, \beta_2'} P(Z_1, N_1, \beta_2, J, t)]. \end{aligned}$$

□ Configuration probability distribution :

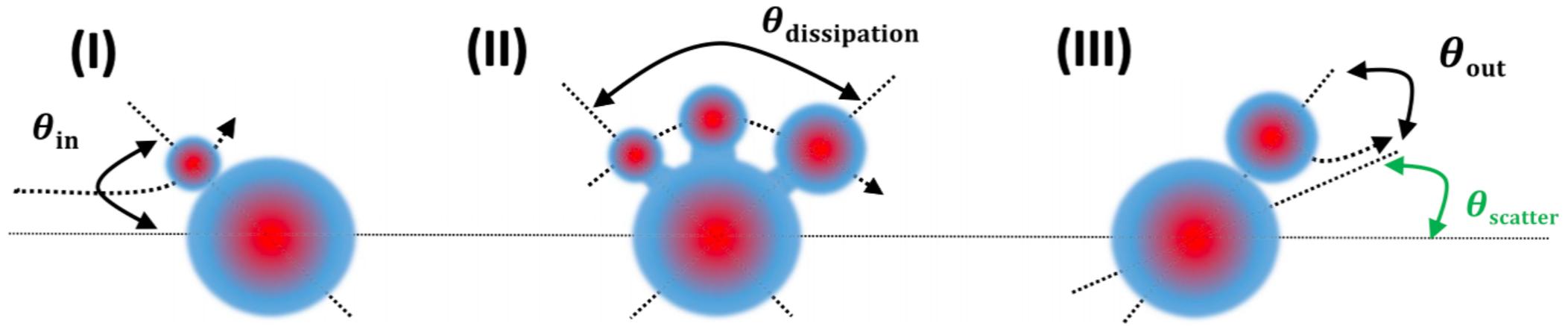
PLF: projectile like fragment
TLF: target like fragment



□ Statistical model:



双核模型 - 角分布



$$\theta_{\text{scatter}} = \pi - \theta_{\text{in}} - \theta_{\text{dissipation}} - \theta_{\text{out}},$$

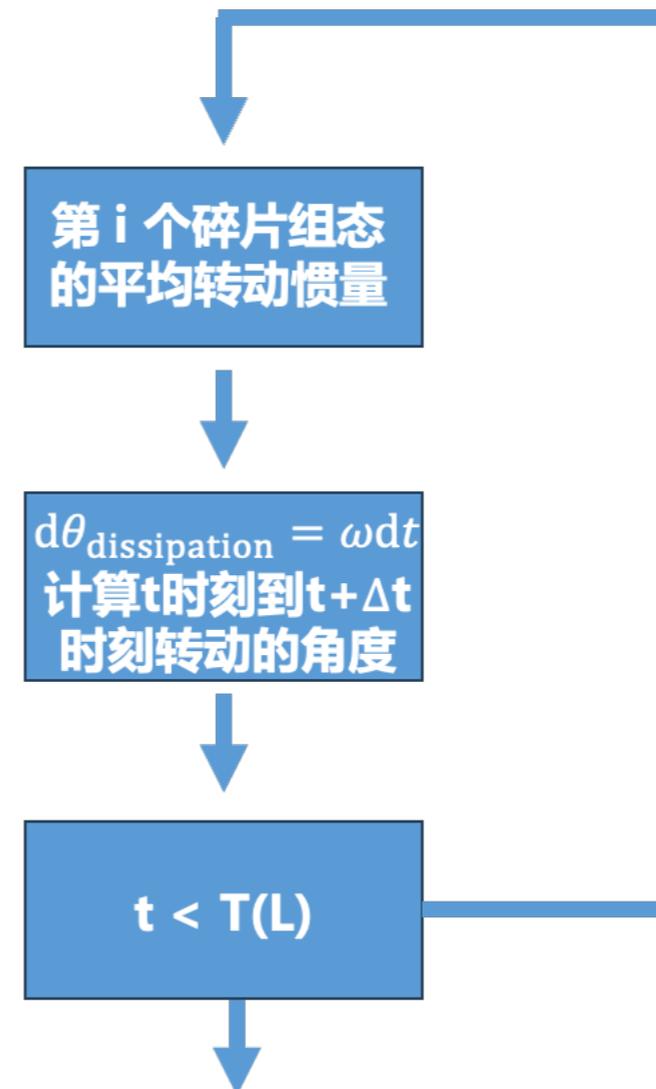
◆ Under the Center of mass frame

$\theta_{\text{dissipation}}$ 由反应时间与转动速度决定

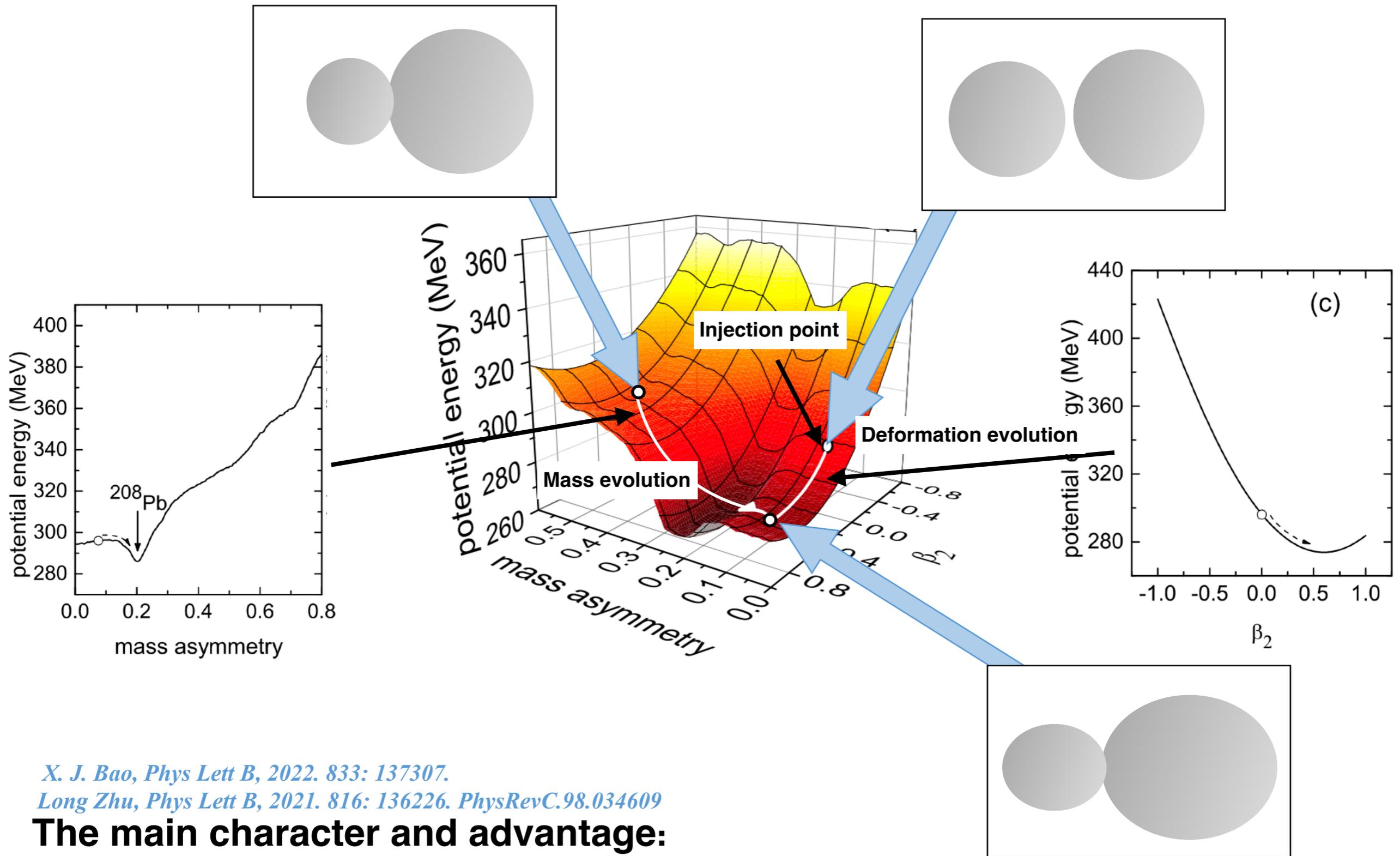
$$\theta_{\text{dissipation}} = \omega_{\text{DNS}} * t_s = \int^{t_s} \frac{J(t)\hbar}{\langle I \rangle} dt.$$

- 考虑角动量耗散的弛豫行为
- 考虑不同碎片组态的刚体转动惯量

PHYSICAL REVIEW RESEARCH 5, L022021 (2023)



双核模型 - 形变自由度



X. J. Bao, Phys Lett B, 2022. 833: 137307.

Long Zhu, Phys Lett B, 2021. 816: 136226. PhysRevC.98.034609

The main character and advantage:

- Introduce to the evolution of the deformation degree of freedom.
- Condition of equal forces of deformation between two nuclei.

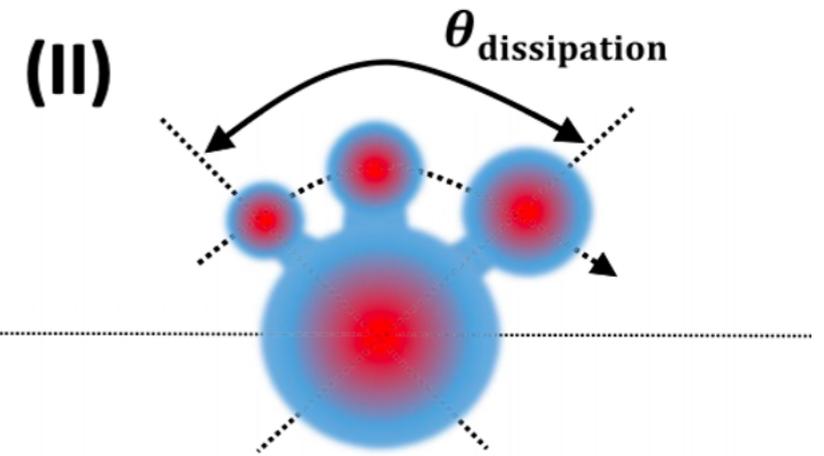
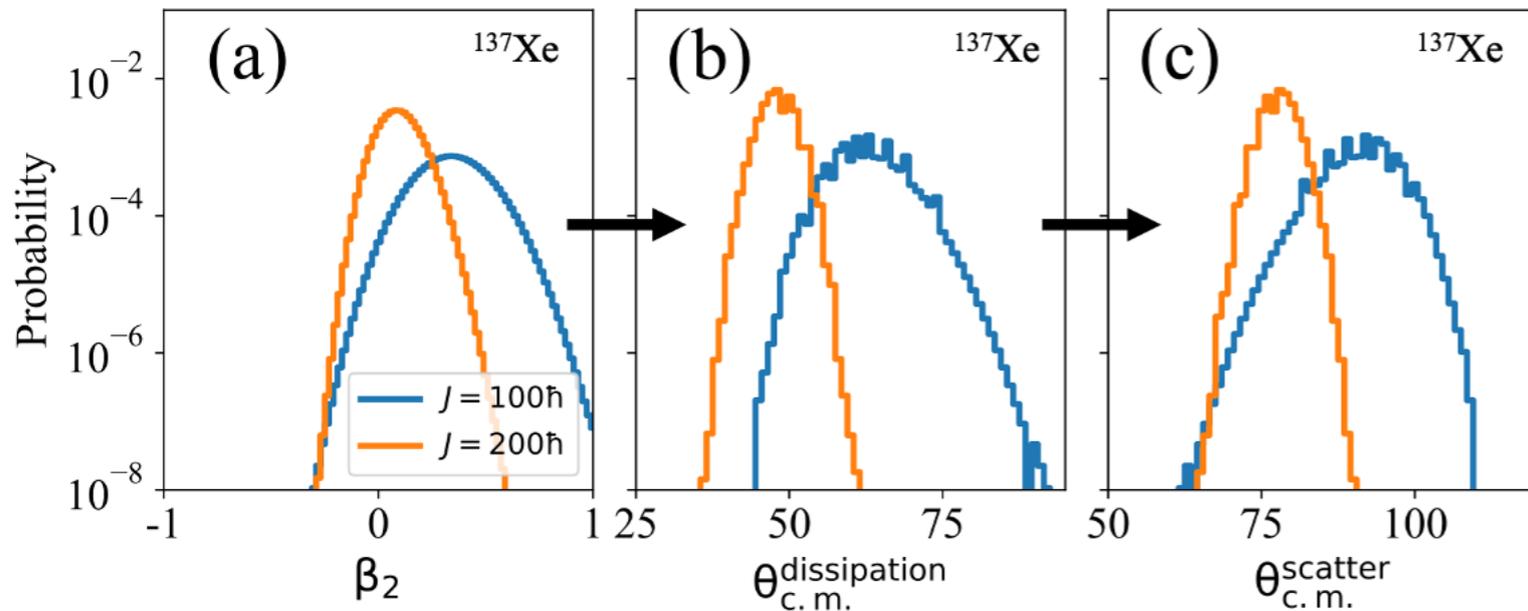
双核模型 - 角分布

$^{136}\text{Xe} + ^{208}\text{Pb} \rightarrow ^{137}\text{Xe} @ E_{\text{c.m.}} = 526\text{MeV}$

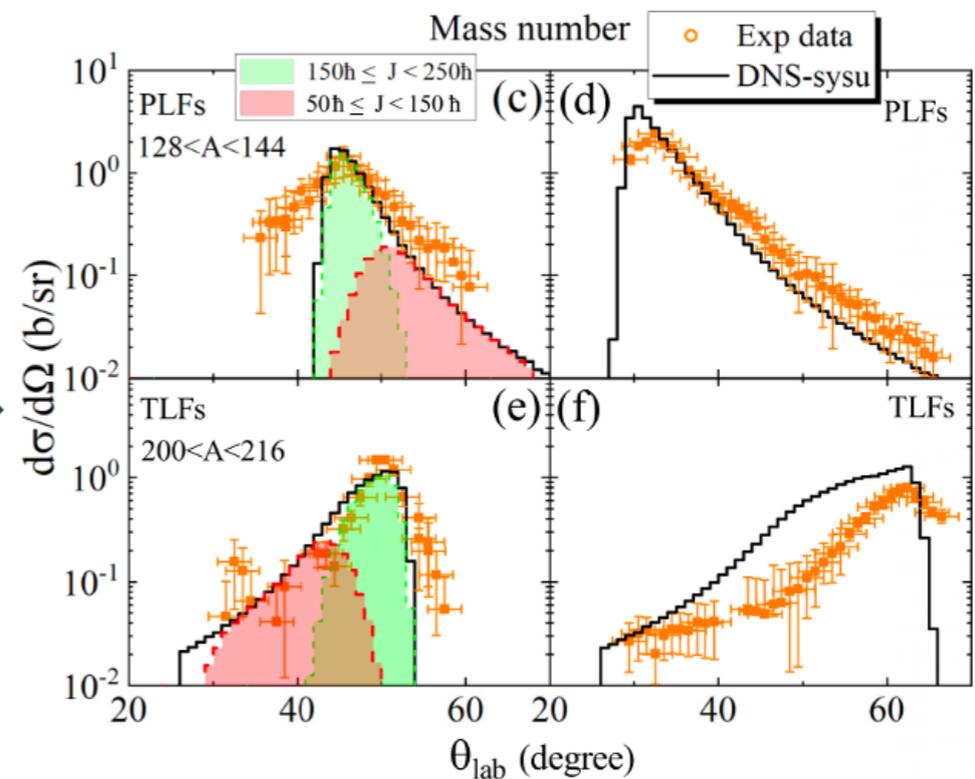
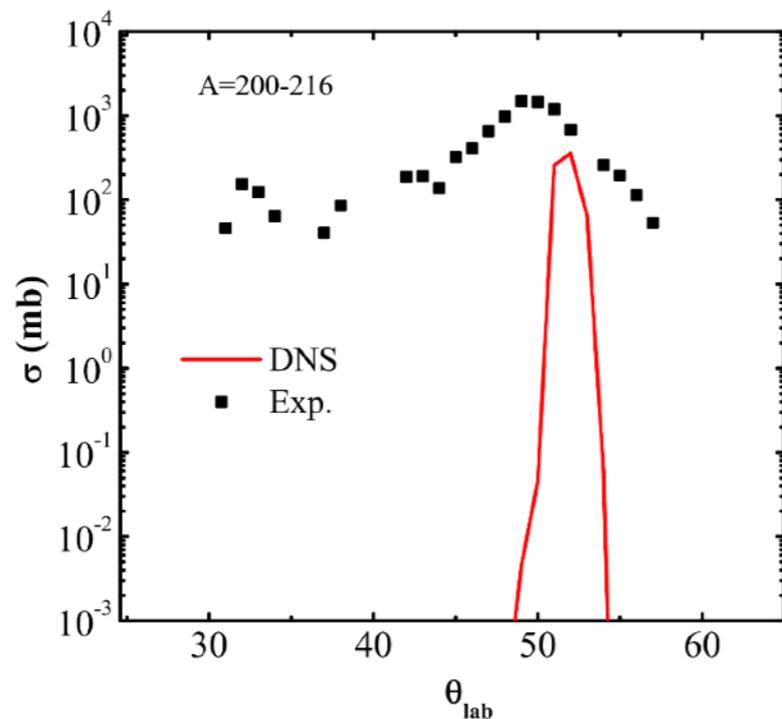
四级形变的概率

耗散角的概率

出射角的概率



■ 在双核模型中 碎片形变的自由度演化的引入 可以自洽地增加碎片出射角度的涨落



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■ 总结与展望

双核模型 - 角分布

Chinese Physics C Vol. 48, No. 10 (2024) 104103

Angular distribution of products in multinucleon transfer reactions*

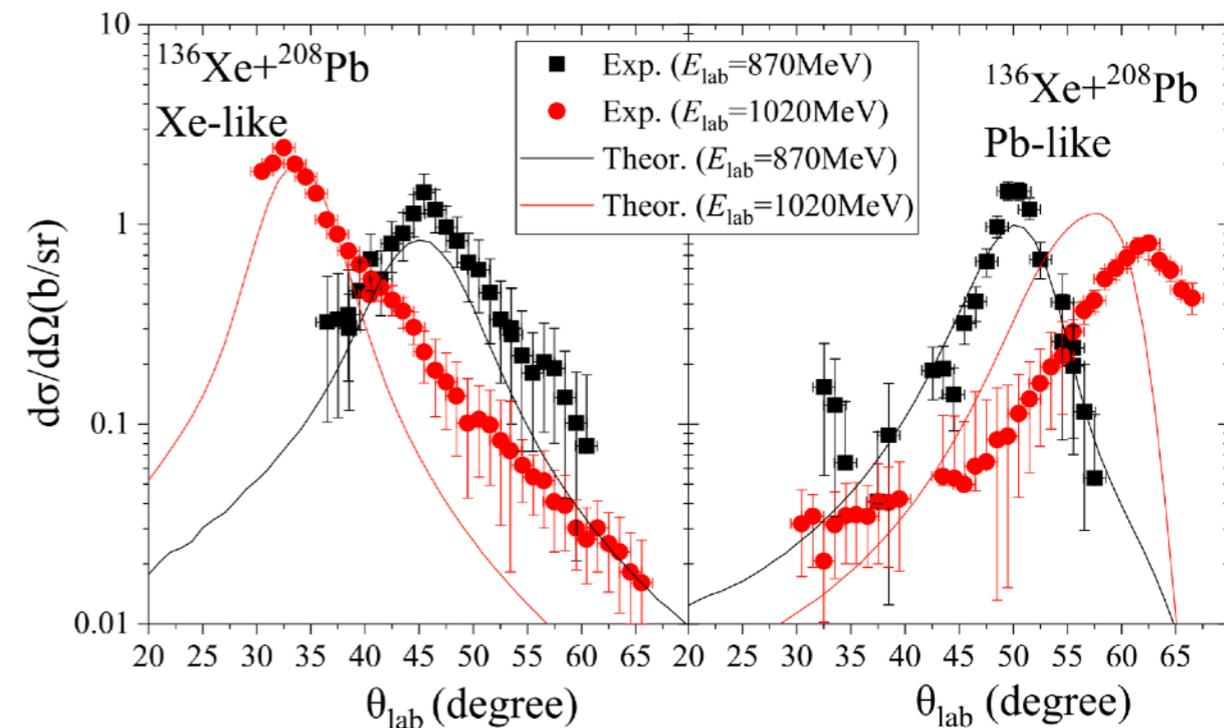
Xian-Kai Le (乐先凯)¹  Kuan Fan (范宽)¹ Feng-Zhu Xing (邢凤竹)¹
Shu-Qing Guo (郭树青)^{1†} Nan Wang (王楠)^{1,2‡} 

¹College of Physics and Optoelectronic Engineering, Shenzhen University, Shenzhen 518060, China

²Shenzhen Key Laboratory of Nuclear and Radiation Safety, Shenzhen 518060, China

$$\frac{d\sigma}{d\Theta} = 2\pi \sum_i \frac{b_i}{(2\pi\sigma_\Theta^2)^{1/2}} \exp \left\{ -\frac{[\Theta - \bar{\Theta}(b_i)]^2}{2\sigma_\Theta^2(b_i)} \right\} \cdot \Delta b,$$

$$\sigma_\Theta^2(b_i, t) = 2C^2 T \frac{J_{\text{int}}}{J_{\text{rel}} \cdot J_{\text{tot}}} \tau_l^2 \cdot \left[\frac{t}{\tau_l} - \frac{1}{2} \exp \left(-\frac{2}{\tau_l} \right) + 2 \exp \left(-\frac{t}{\tau_l} \right) - \frac{3}{2} \right],$$



两参数 半经验公式

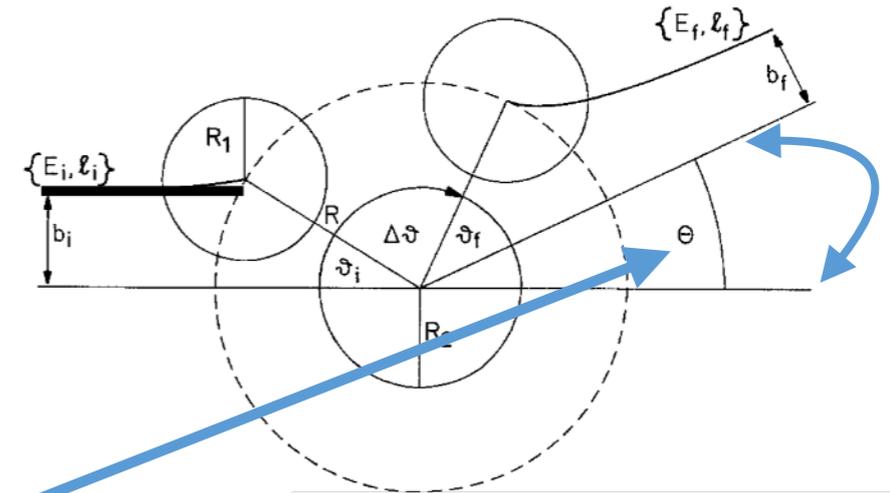
Z. Physik A 284, 209–216 (1978)

**Zeitschrift
für Physik A**
© by Springer-Verlag 1978

Analysis of Relaxation Phenomena in Heavy-Ion Collisions*

G. Wolschin and W. Nörenberg
Institut für theoretische Physik der Universität
and Max-Planck-Institut für Kernphysik, Heidelberg

Received June 27, 1977



$$\theta_{\text{scatter}} = \pi - \theta_{\text{in}} - \theta_{\text{dissipation}} - \theta_{\text{out}}$$

Z. Physik A 284, 209-216 (1978)

• 两参数的偏转函数:

$$\Theta(l_i) = 2 \arctan \frac{Z_p Z_T e^2}{2E_{c.m.} b} - \beta \Theta_C^{gr} \frac{l_i}{l_{gr}} \left(\frac{\delta}{\beta} \right)^{l_i/l_{gr}}$$

“库伦力”偏转

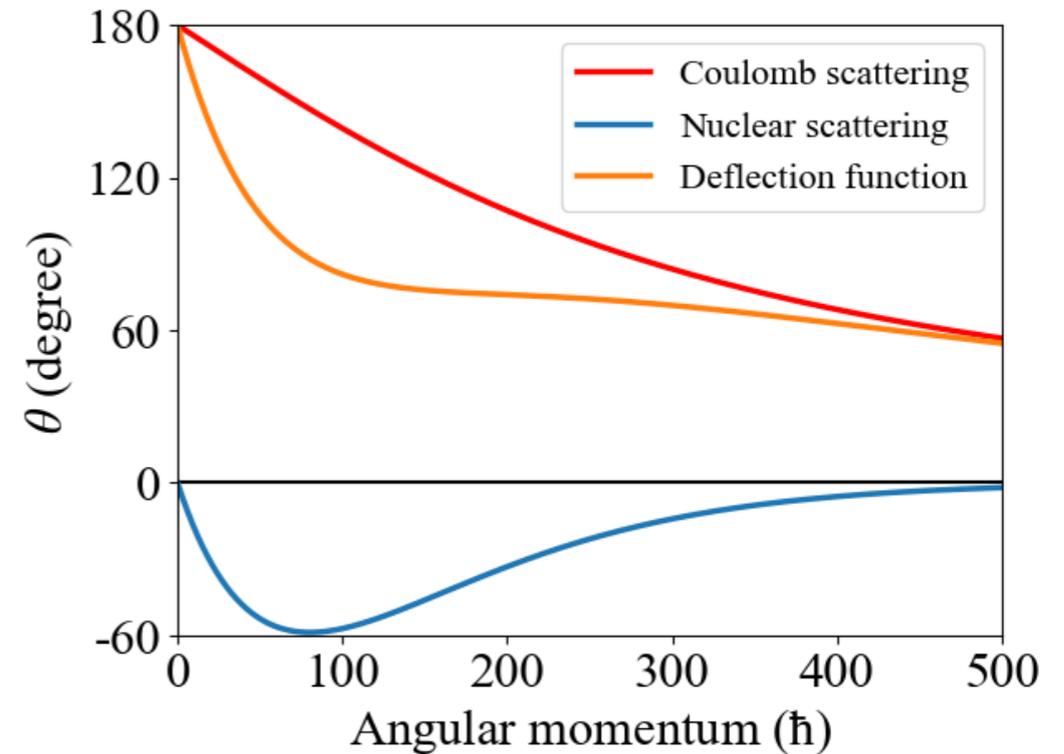
“核力”偏转

入射能: $E_{c.m.}$

擦边角: Θ^{gr}

碰撞参数: b

擦边角动量: l_{gr}



半经验公式 · 角分布

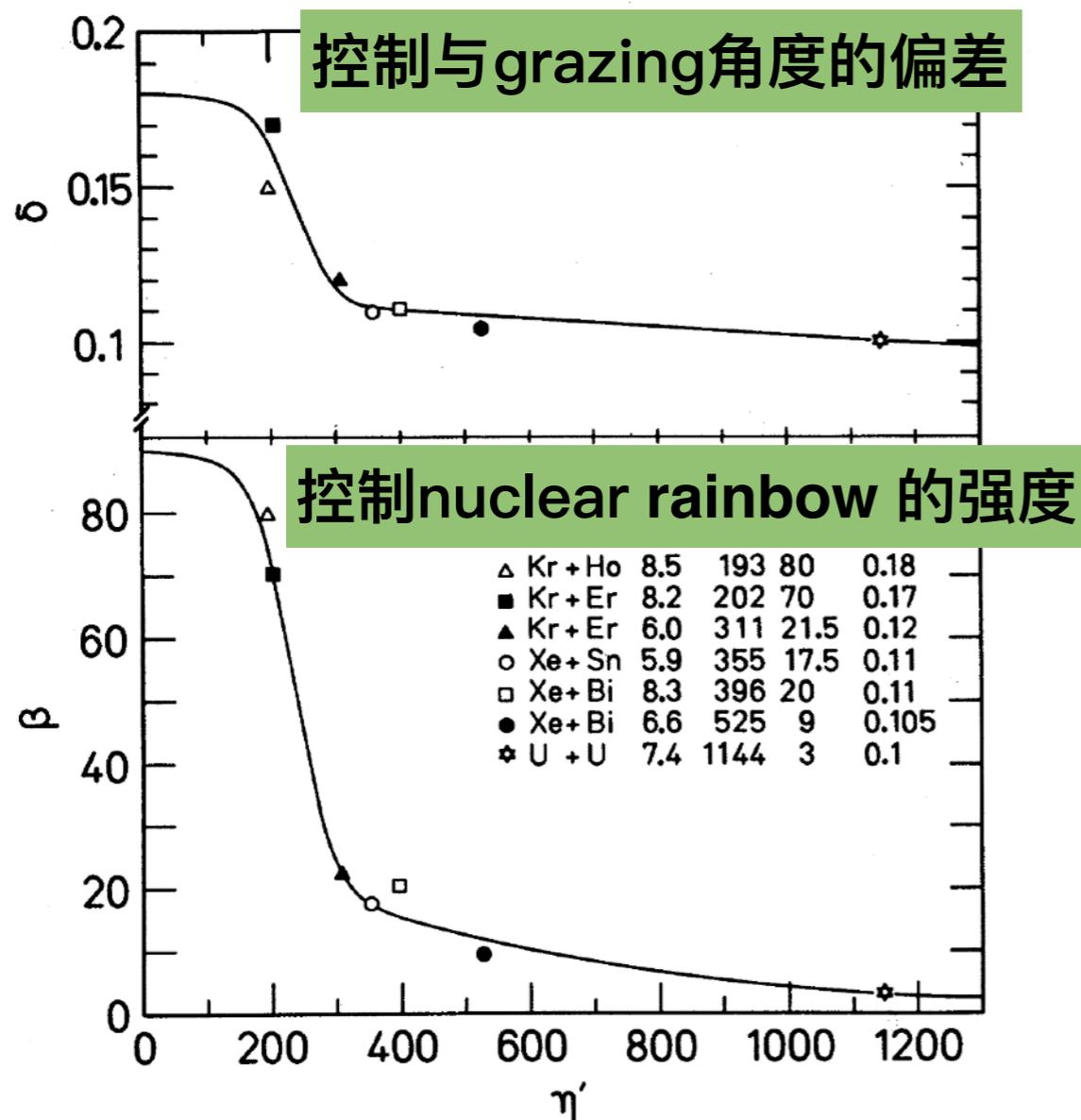


FIG. 1. Parameters β, δ determining the mean deflection angle $\langle \theta(l) \rangle$ as functions of $\eta' = Z_1 Z_2 e^2 / v'$. Symbols represent fits to the angular distributions, the solid curves the analytical expressions given in the text.

过去, 可以使用偏转函数去估计碎片的出射角度

Li, J.Q, et al, PhysRevC.27.590

$$\Theta(J_i) = \underbrace{\Theta_C(J_i)}_{\text{库伦项}} + \underbrace{\Theta_N(J_i)}_{\text{核力项}}$$

库伦项

核力项

$$\Theta(J_i)_C = 2 \arctan \frac{Z_p Z_t e^2}{2E_{c.m.} b}$$

$$\Theta(J_i)_N = -\beta \Theta_C^{gr}(J_i) \frac{J_i}{J_{gr}} \left(\frac{\delta}{\beta} \right)^{J_i/J_{gr}}$$

The parameters β and δ depend on the amount of focussing observed in the angular distribution. The deviation from the Coulomb trajectory near the grazing angle is determined by δ . Large values of β correspond to negative angle scattering which is frequently encountered in reactions with $E/B \gtrsim 1.7$ [3] or $\eta' = \alpha/(\hbar v') \lesssim 200$ for a modified Sommerfeld parameter η' with v' being the relative velocity at the interaction radius [2]. Small values of β describe strong focussing where the nuclear rainbow of the deflection function (2.10) moves close to or coincides with the Coulomb rainbow. Possible physical reasons for the change in focussing as a function of E/B or η' have been discussed in [2, 3].

两参数 半经验公式

两参数的偏转函数：

$$\Theta(l_i) = 2 \arctan \frac{Z_p Z_T e^2}{2E_{c.m.} b} - \beta \Theta_C^{gr} \frac{l_i}{l_{gr}} \left(\frac{\delta}{\beta} \right)^{l_i/l_{gr}}$$

其中， β 和 δ 通过实验数据拟合，与反应体系、反应能量相关

对实验数据峰值描述比较好，对无法描述实验数据的展宽

$$\frac{d\sigma}{d\Theta} = \frac{2\pi}{k^2} \sum_n l_n \left| \frac{dl}{d\Theta} \right|_{l=l_n}$$

$$\beta = 75f(\eta') + 15, \quad \eta' < 375$$

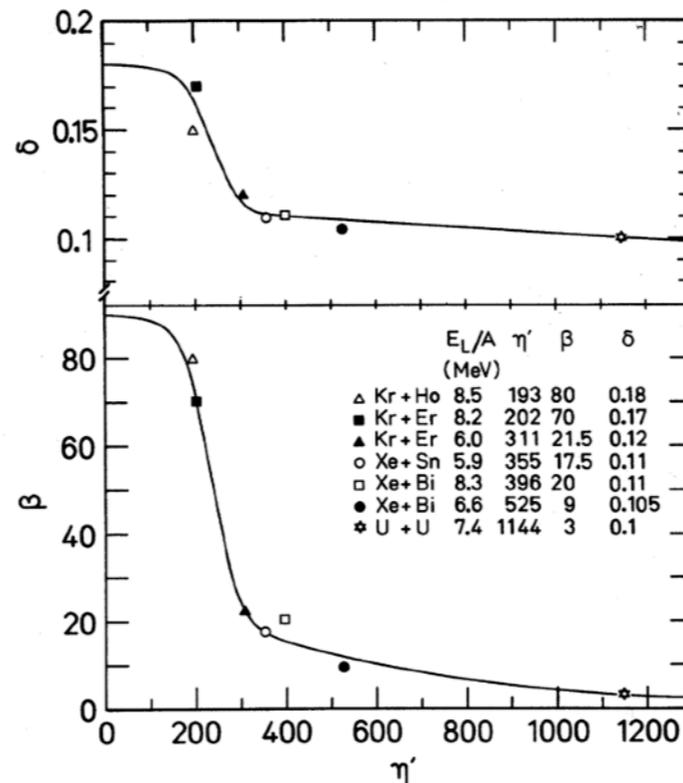
$$= 36 \exp[-2.17 \times 10^{-3} \eta'], \quad \eta' \geq 375,$$

$$\delta = 0.07f(\eta') + 0.11, \quad \eta' < 375$$

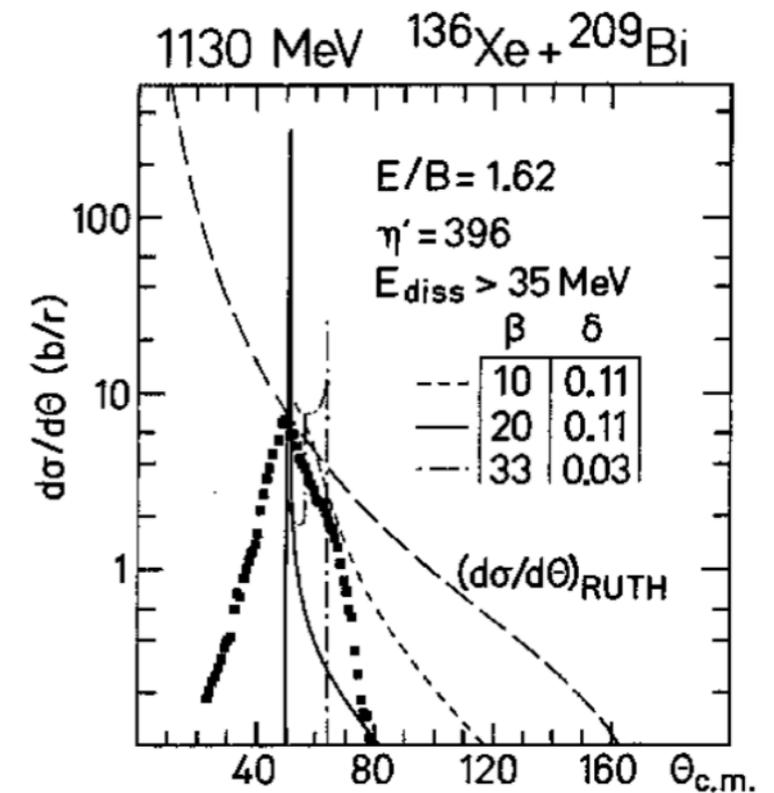
$$= 0.117 \exp[-1.34 \times 10^{-4} \eta'], \quad \eta' \geq 375,$$

where

$$f(\eta') = \left[1 + \exp \left[\frac{\eta' - 235}{32} \right] \right]^{-1}$$



Phys. Rev. C. 27, 590 (1983)



Z. Physik A 290, 47-55 (1979)

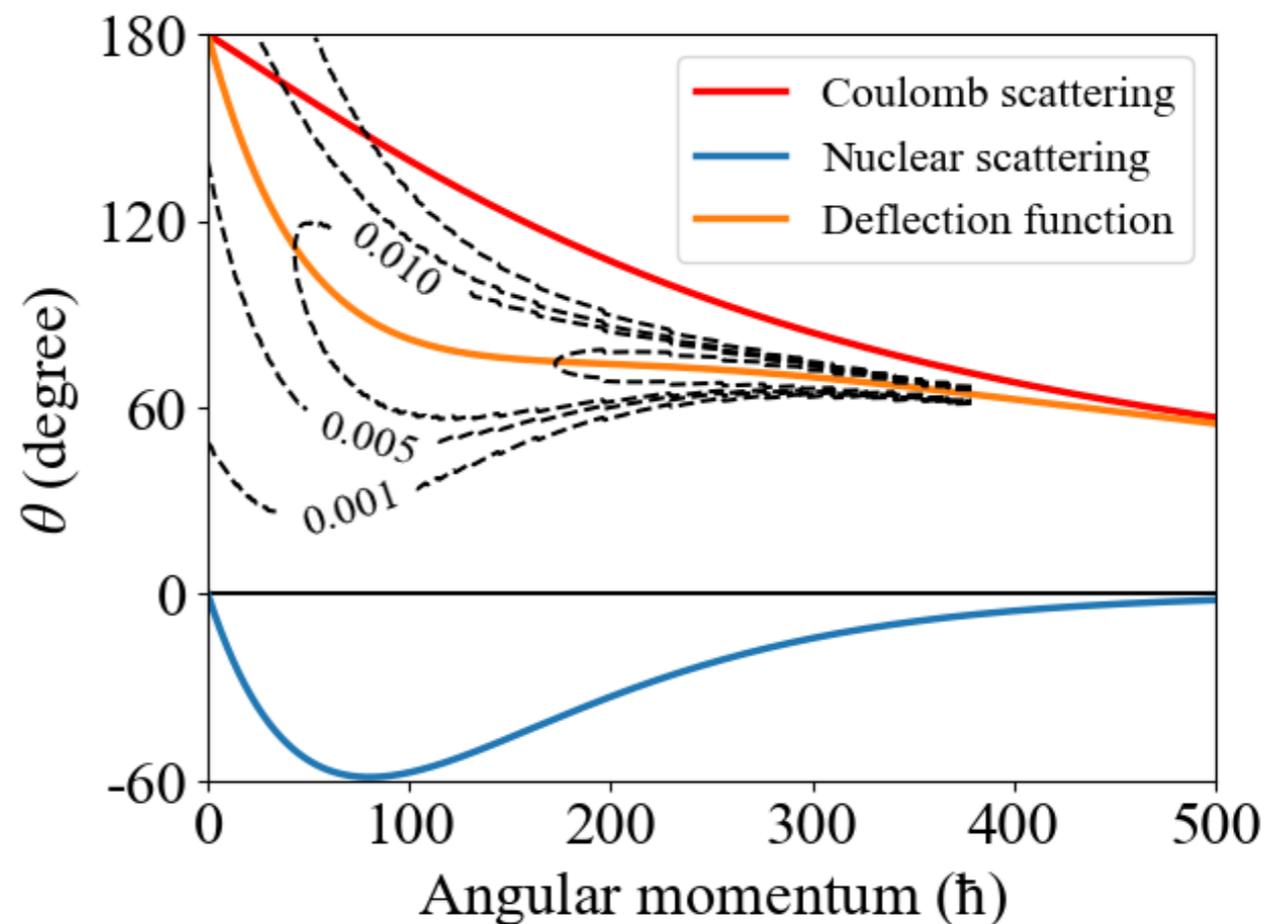
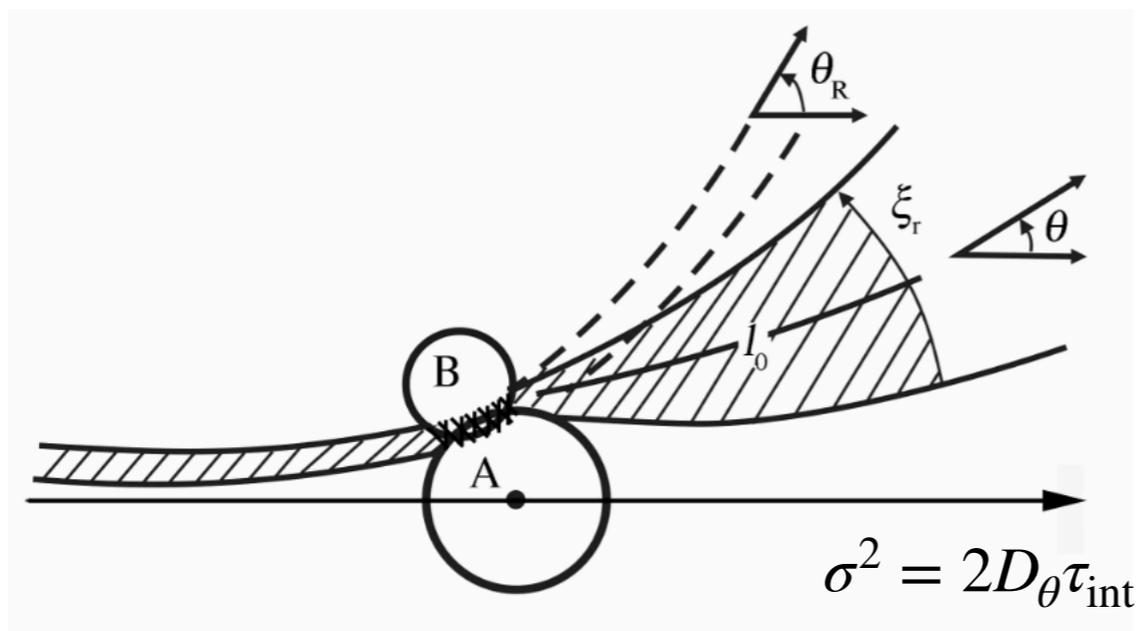
三参数 半经验公式 · 角分布

三参数的偏转函数：

$$\Theta(l_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\Theta - \bar{\Theta})^2}{2\sigma^2}\right) \left(2 \arctan \frac{Z_p Z_T e^2}{2E_{c.m.} b} - \beta \Theta_C^{gr} \frac{l_i}{l_{gr}} \left(\frac{\delta}{\beta}\right)^{l_i/l_{gr}}\right)$$

高斯展宽

引入高斯分布，考虑碰撞过程当中涨落效应

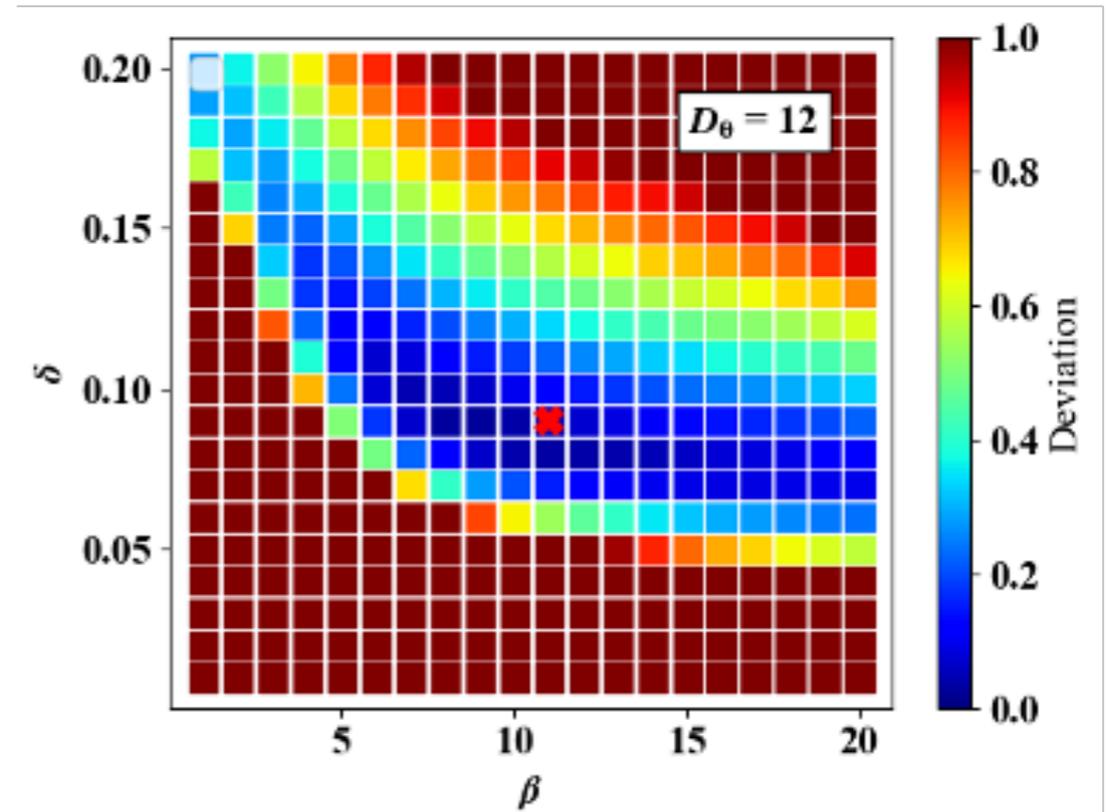


其中， σ 、 β 和 δ 通过实验数据拟合，与反应体系、反应能量，反应时间相关

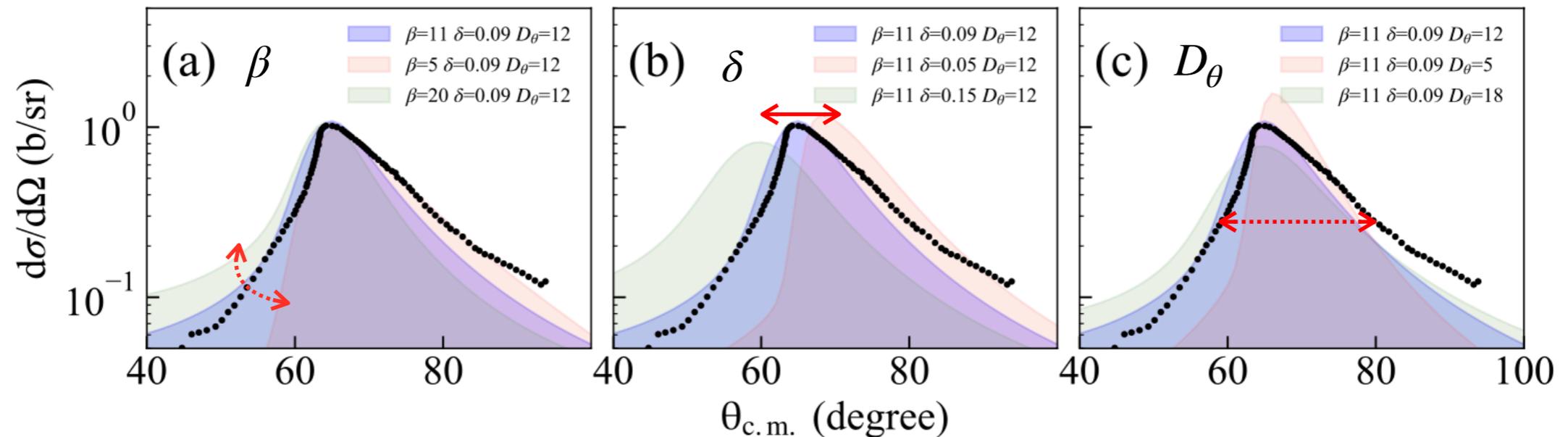
最优参数选取

- 为了定量描述计算与实验值的偏离程度，我们通过引入平均偏差：

$$\mathfrak{D} = \frac{1}{n} \sum_{i=1}^n \left[\log \left(\frac{\sigma_{\text{th}}(\theta_i)}{\sigma_{\text{exp}}(\theta_i)} \right) \right]^2$$



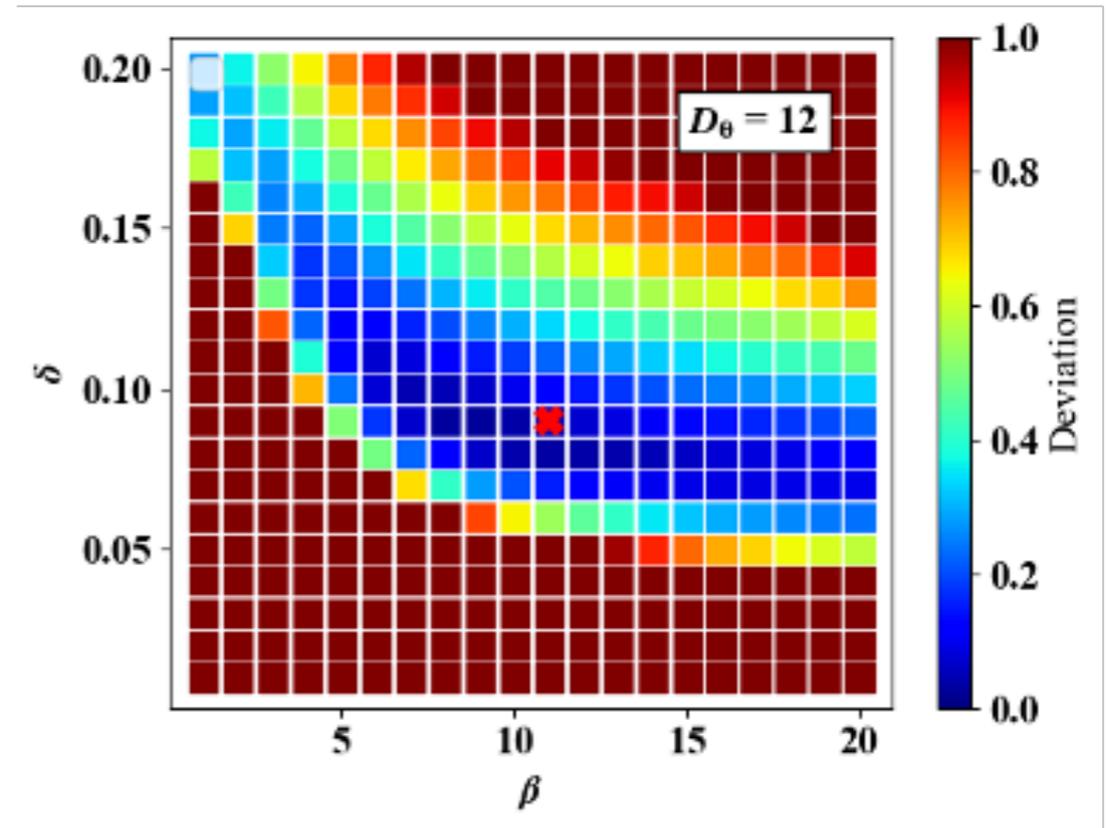
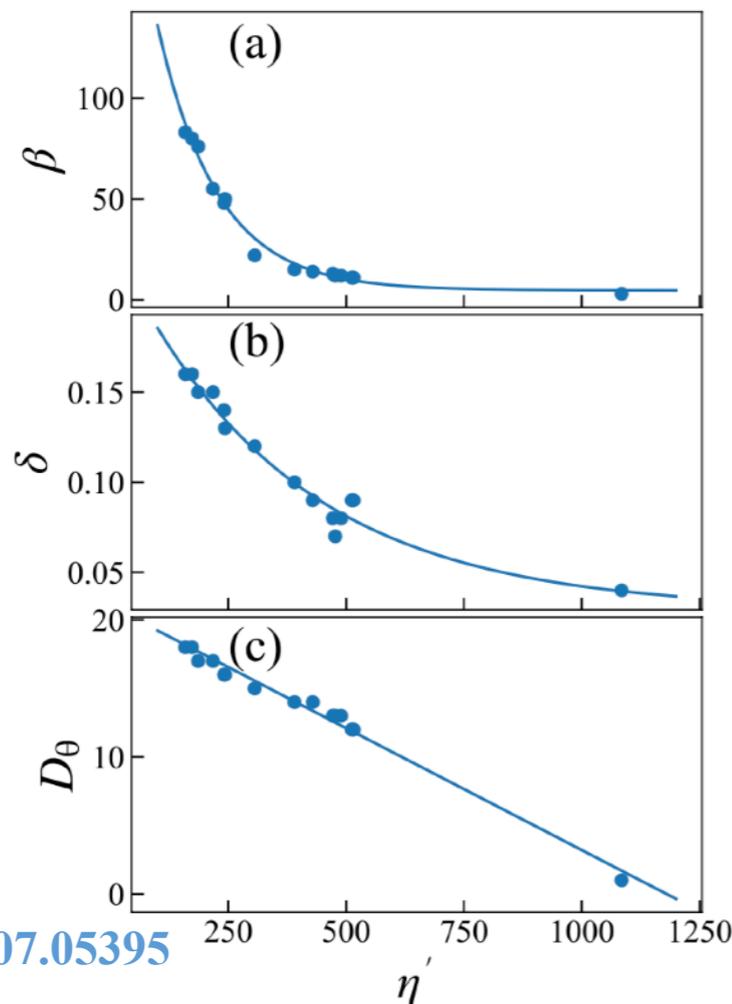
- 参数定量分析：



最优参数选取

- 为了定量描述计算与实验值的偏离程度，我们通过引入平均偏差：

$$\mathfrak{D} = \frac{1}{n} \sum_{i=1}^n \left[\log \left(\frac{\sigma_{\text{th}}(\theta_i)}{\sigma_{\text{exp}}(\theta_i)} \right) \right]^2$$



$$\beta(\eta) = 4.7 + 81.2 * \exp[-(\eta - 160.8)/126.5]$$

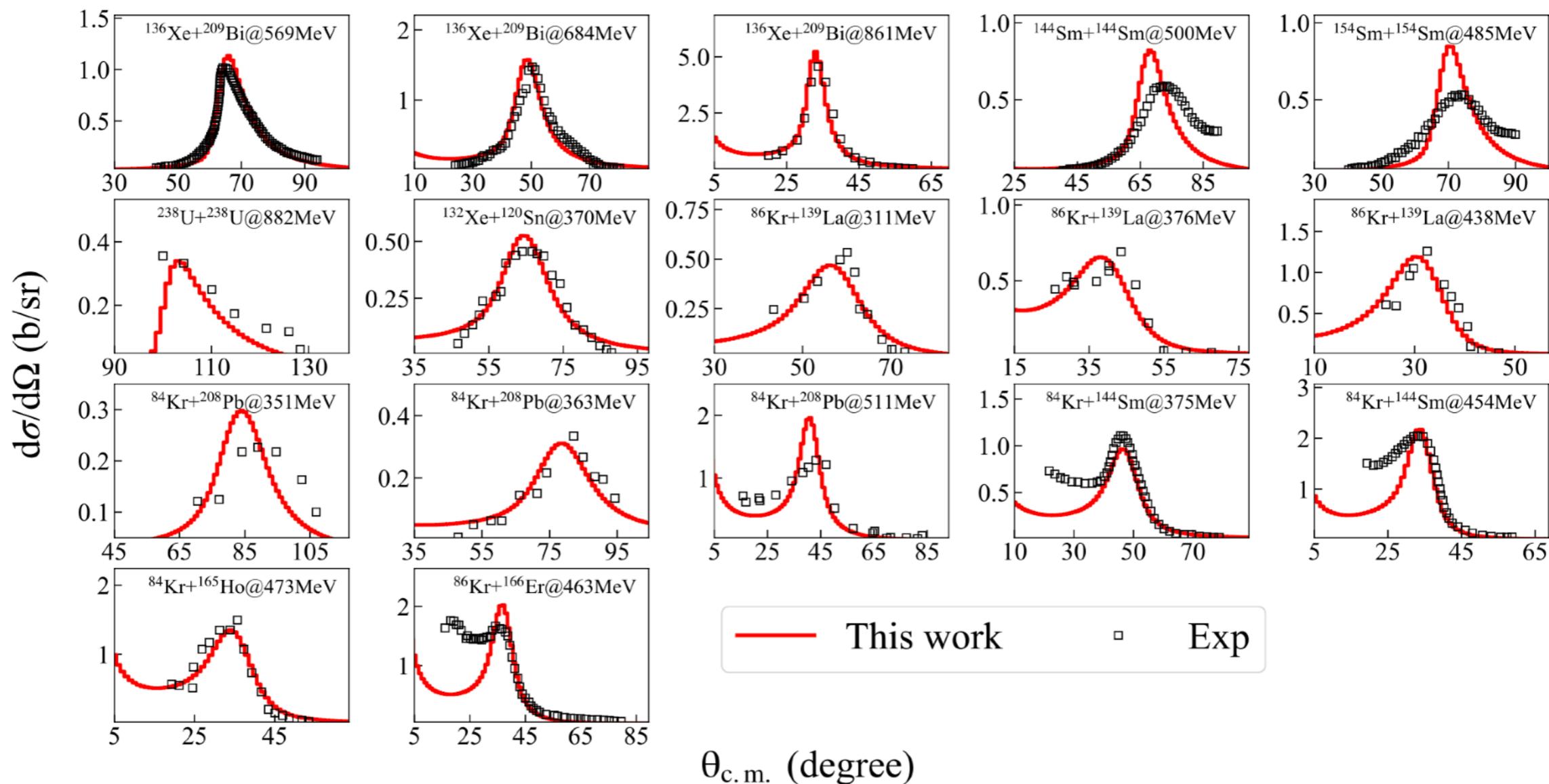
$$\delta(\eta) = 0.029 + 0.127 * \exp[-(\eta - 176.3)/364]$$

$$D_\theta(\eta) = 21 - 0.0178\eta$$

研究内容

三参数的偏转函数：

$$\Theta(l_i) = \frac{1}{\sigma\sqrt{2\pi}} \exp\left(-\frac{(\Theta - \bar{\Theta})^2}{2\sigma^2}\right) \left(2 \arctan \frac{Z_p Z_T e^2}{2E_{c.m.} b} - \beta \Theta_C^{gr} \frac{l_i}{l_{gr}} \left(\frac{\delta}{\beta}\right)^{l_i/l_{gr}}\right)$$



Bypassing the complex dynamical calculations underlying the MNT process

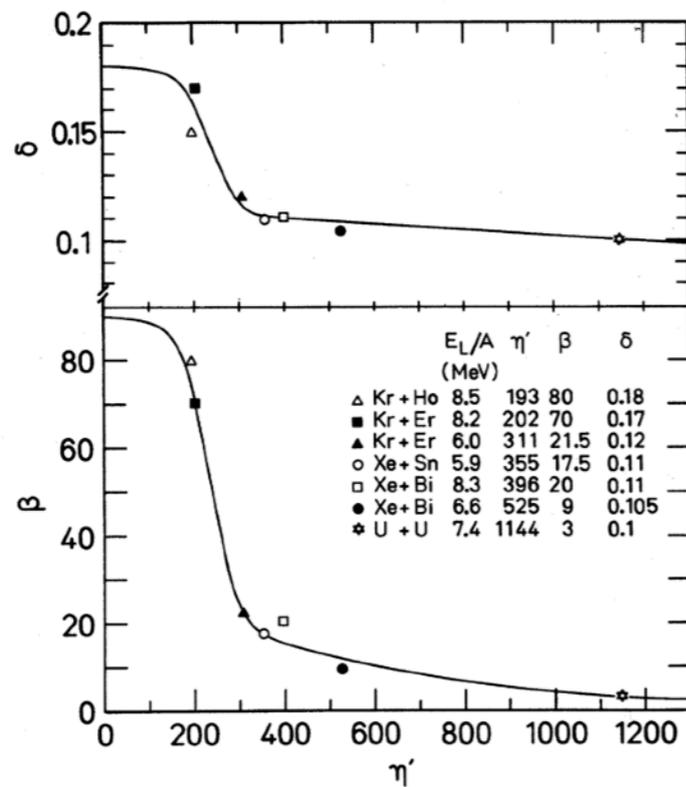
总结与展望

总结:

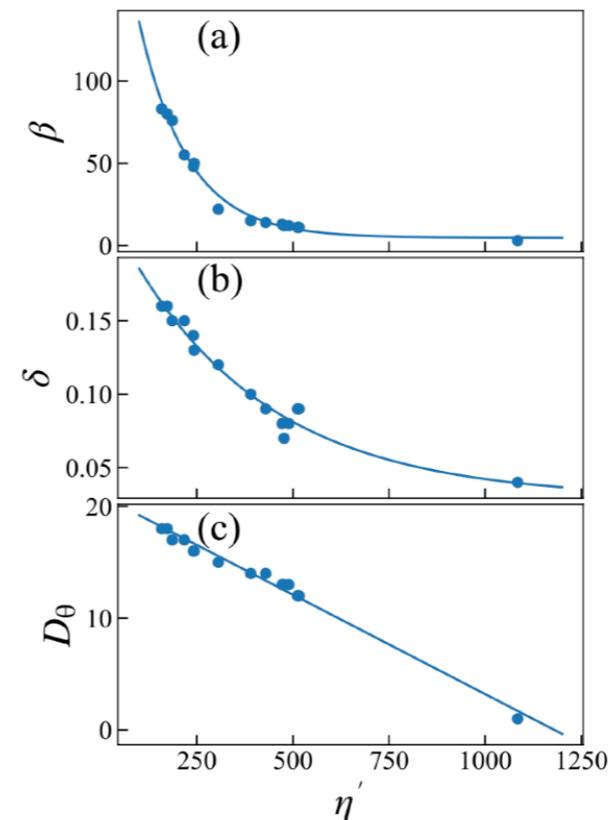
- 在双核模型框架下，通过引入形变自由度，我们可以自洽的描述多核子转移的角分布。
- 我们改进了偏转函数，通过引入唯象的涨落形式，可以用三参数的偏转函数很好的描述多核子转移反应的角分布。

展望:

- 通过加入涨落效应的考虑，新的偏转函数可以被考虑使用计算反应时间（双核模型）



Phys. Rev. C. 27, 590 (1983)



arXiv:2407.05395