

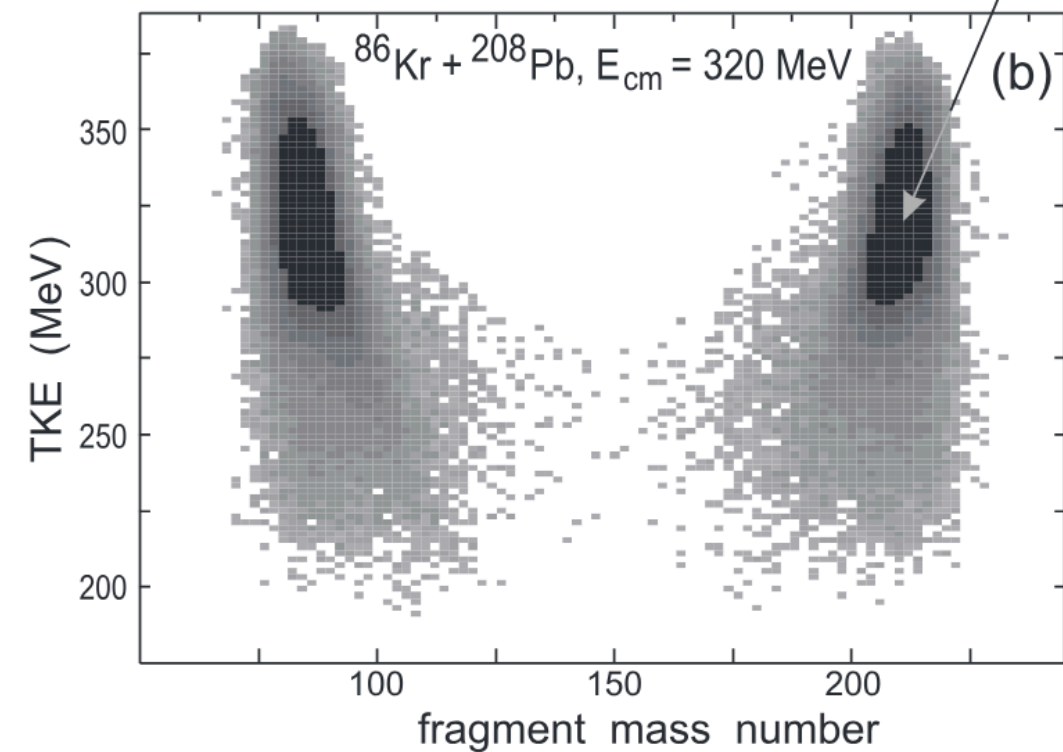
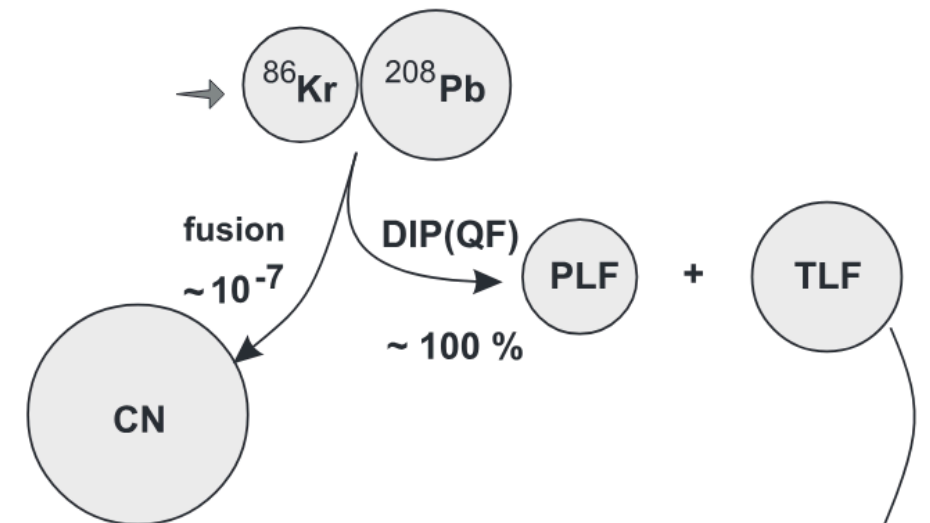
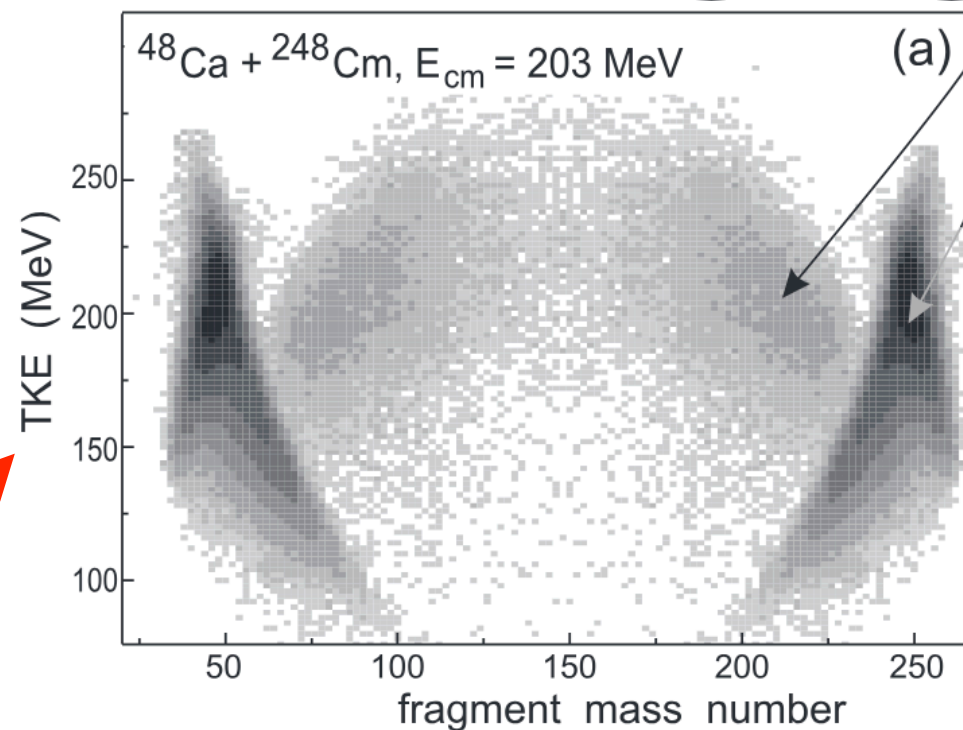
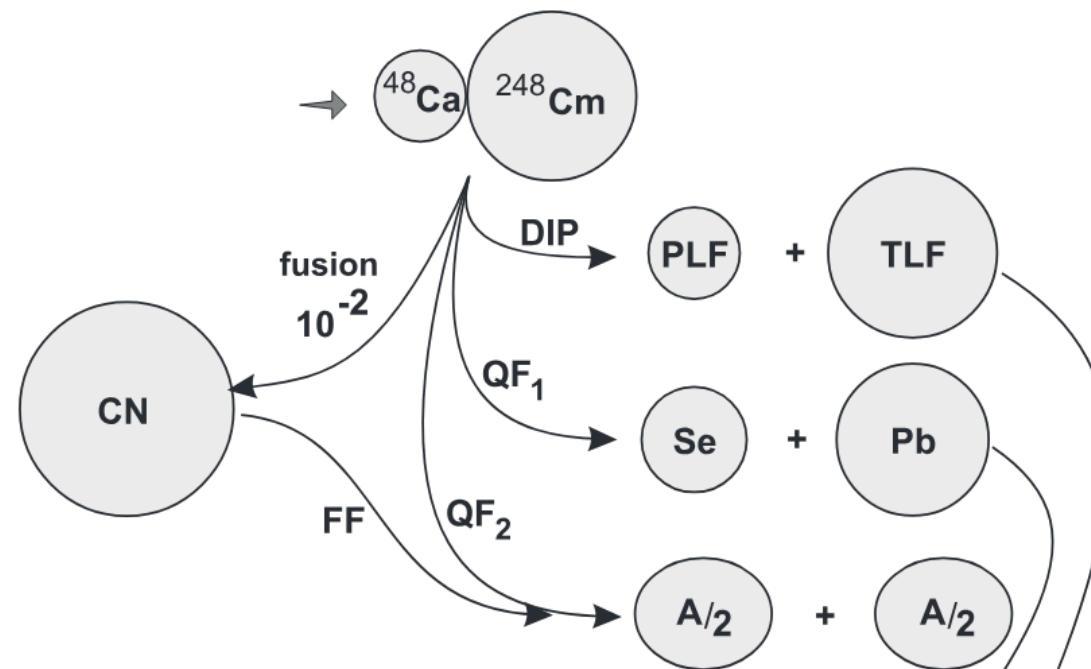
A unified description of low-energy fusion and multinucleon-transfer reactions within the Coupled Master-Langevin model

Speaker: Zehong Liao

Content

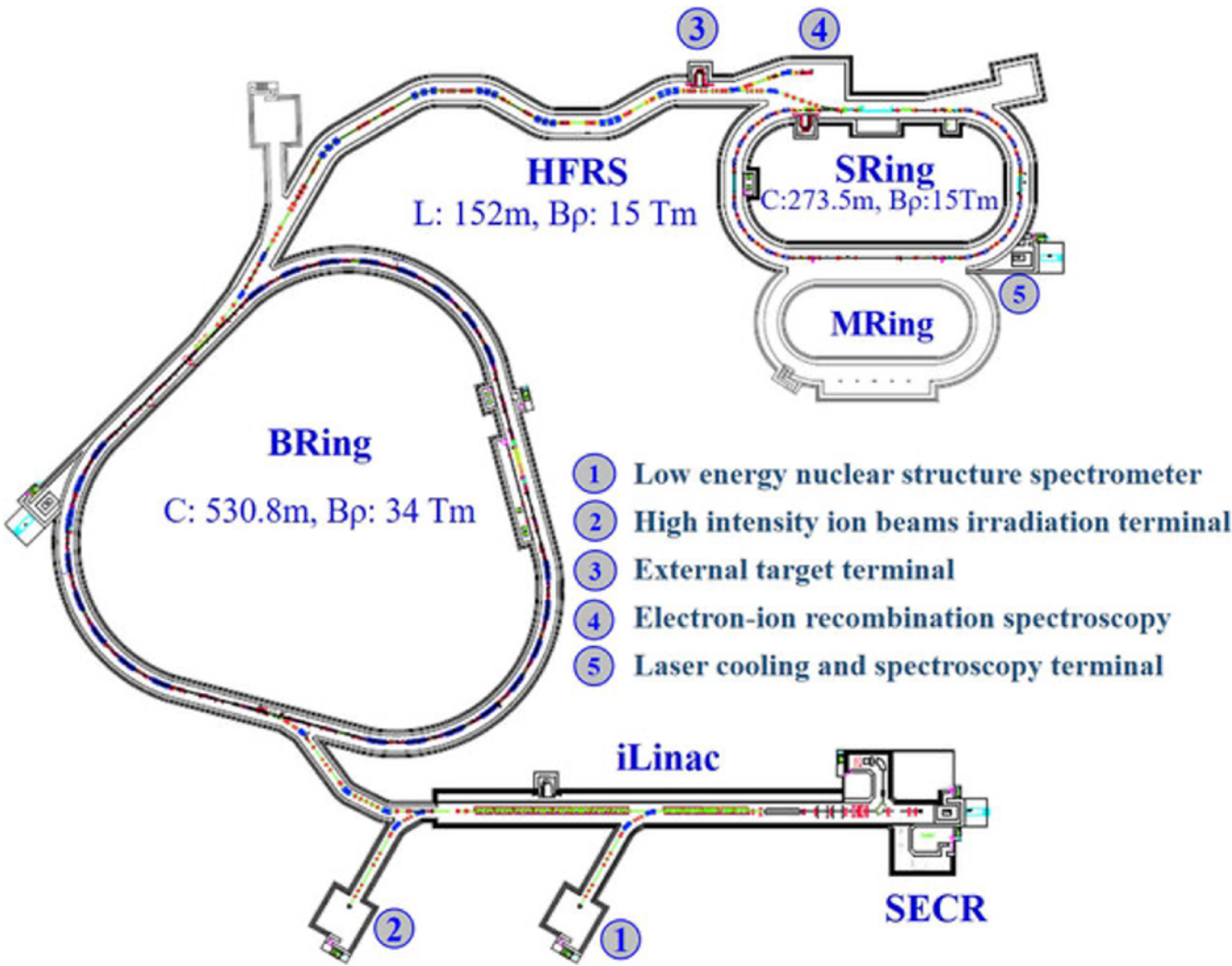
- A. Background**
- B. Degree of freedom**
- C. Potential energy surface**
- D. Equations of motion and nucleon transfer**
- E. Modeling nucleus-nucleus collisions**
- F. Summary**

What can we see from the low energy collision, $E < 10 \text{ MeV/u}$

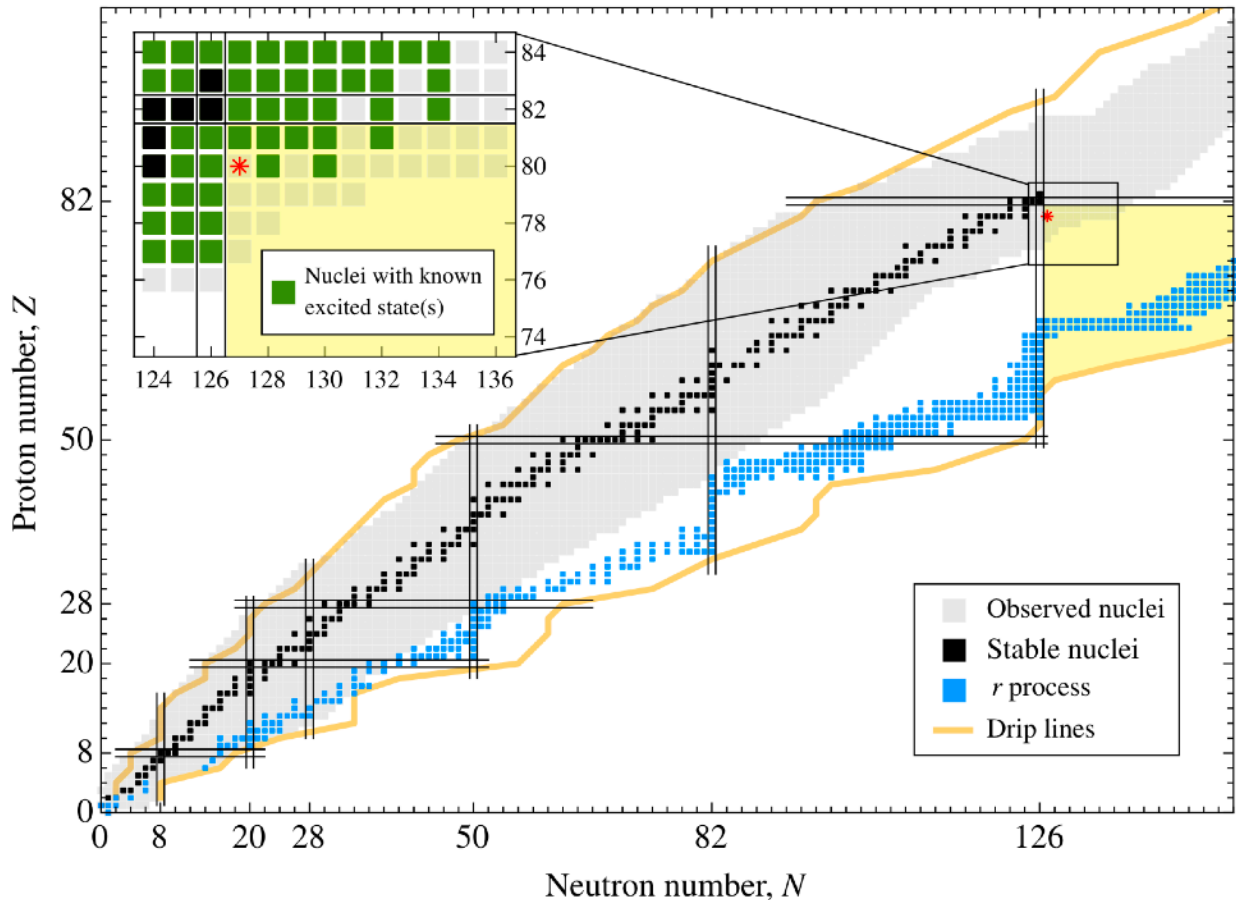


Experiment result: the correlation between the fragment mass and fragment energy

Experiment terminal in HIAF, China

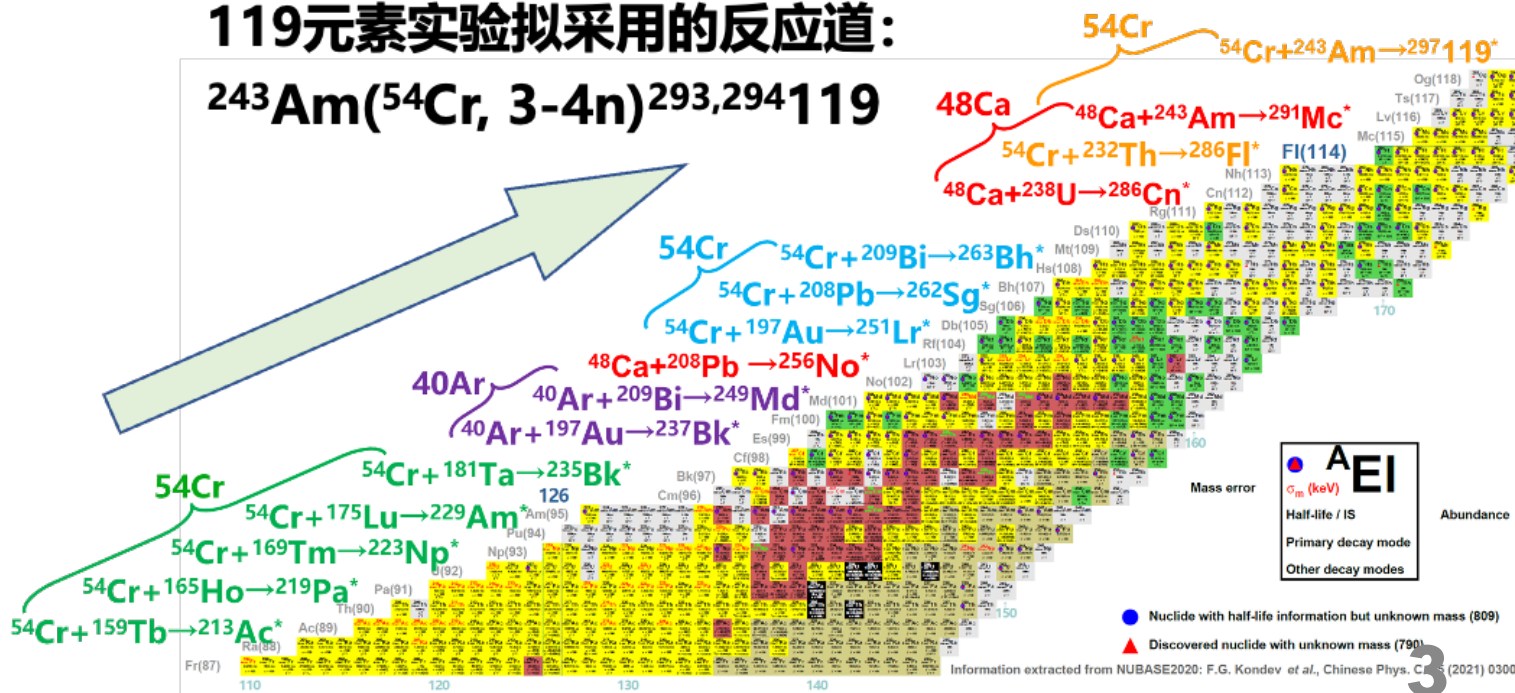
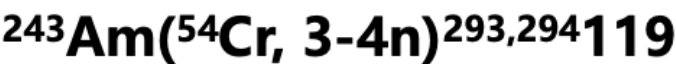


New isotones at N = 126

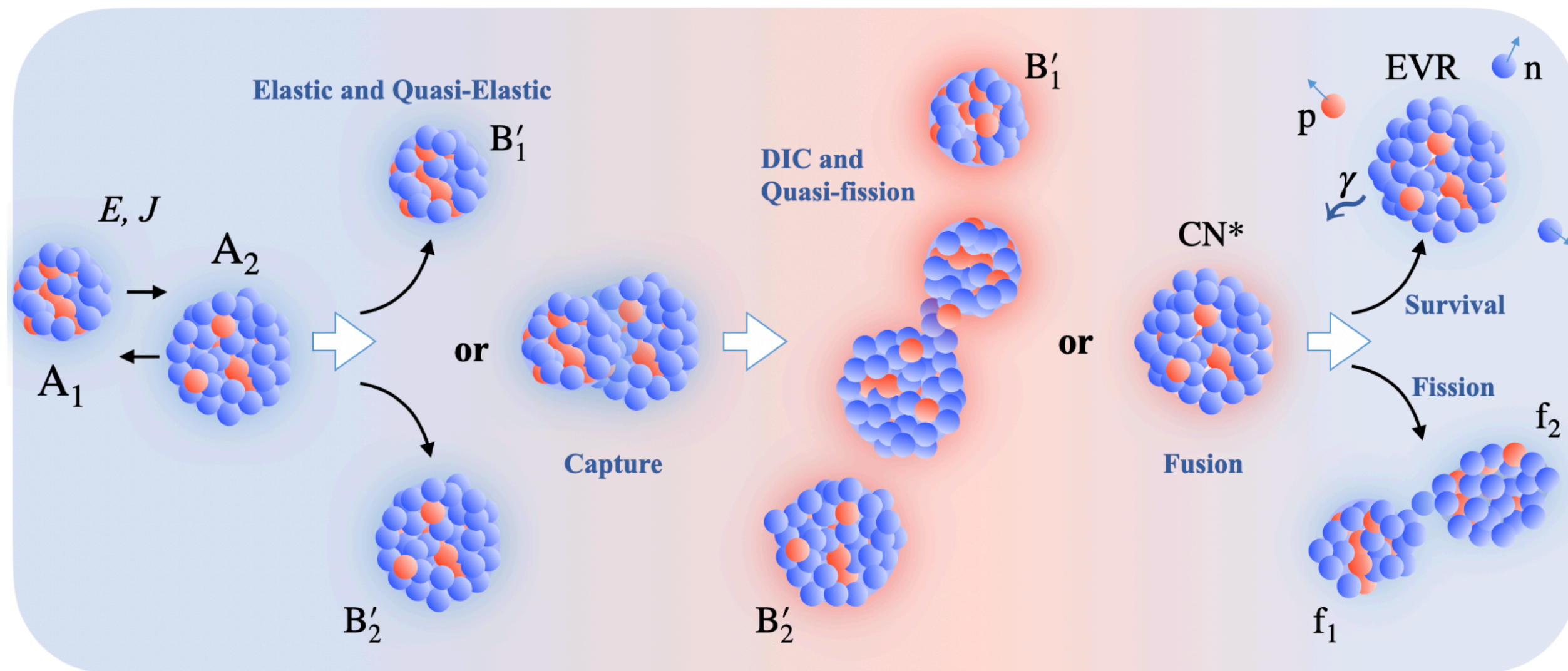


New Superheavy element

119元素实验拟采用的反应道:



Modeling nucleus-nucleus collisions



$$\sigma(Z_f, N_f) = \frac{\pi \hbar^2}{2\mu E_{\text{c.m.}}} \sum_{J=0}^{J_{\text{max}}} (2J+1) P_{\text{cap}}(J) \sum_{Z_1, N_1} P(Z_1, N_1; J^*, E^*) W(Z_1, N_1 \rightarrow Z_f, N_f; J^*, E^*),$$

in generally, this term is the aspect that theorists most want to determine, yet it is also the most difficult to determine.

Different angle to the low energy collision

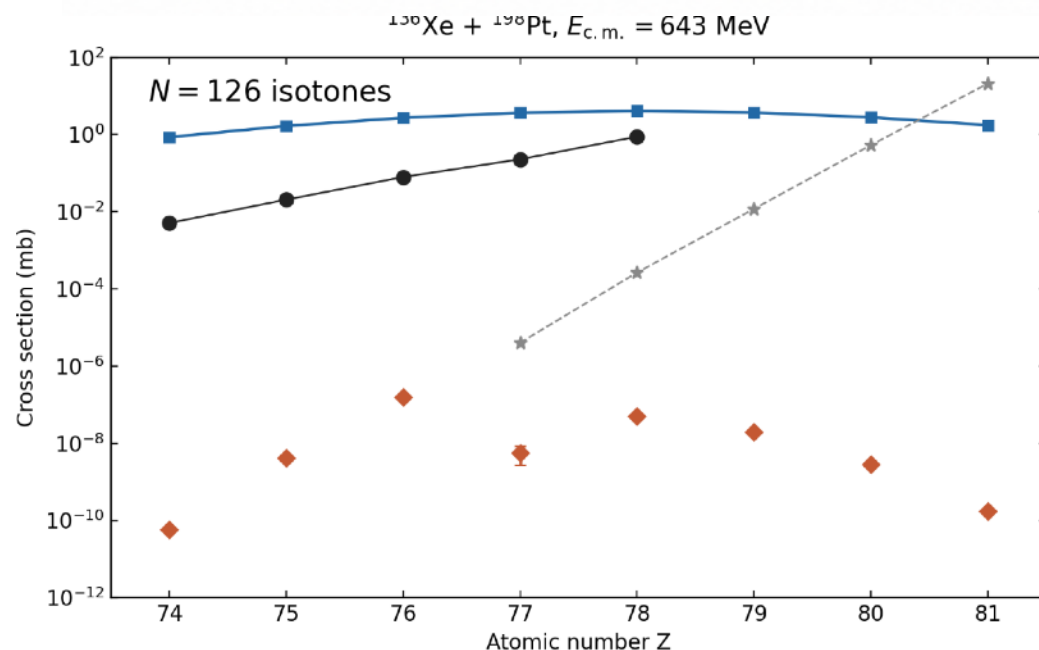
Microscopic — Nucleonic Degrees of Freedom

Starting point:	Self-consistent evolution of individual nucleons in the mean field
Method:	TDHF
Strength:	Microscopic connection between nuclear structure and reaction dynamics
Limitation:	Mean-field approximation underestimates fluctuations and fragment distributions

————— *Nucleonic degrees of freedom* $\longleftrightarrow$ *Collective degrees of freedom* —————

Macroscopic — Collective Degrees of Freedom

Starting point:	Evolution of a few collective variables, e.g. R , δ , η_A , N , Z
Methods:	DNS / Langevin
Strength:	Efficient description of dissipation, diffusion, and broad fragment distributions
Limitation:	Requires modeled PES and transport/transfer coefficients

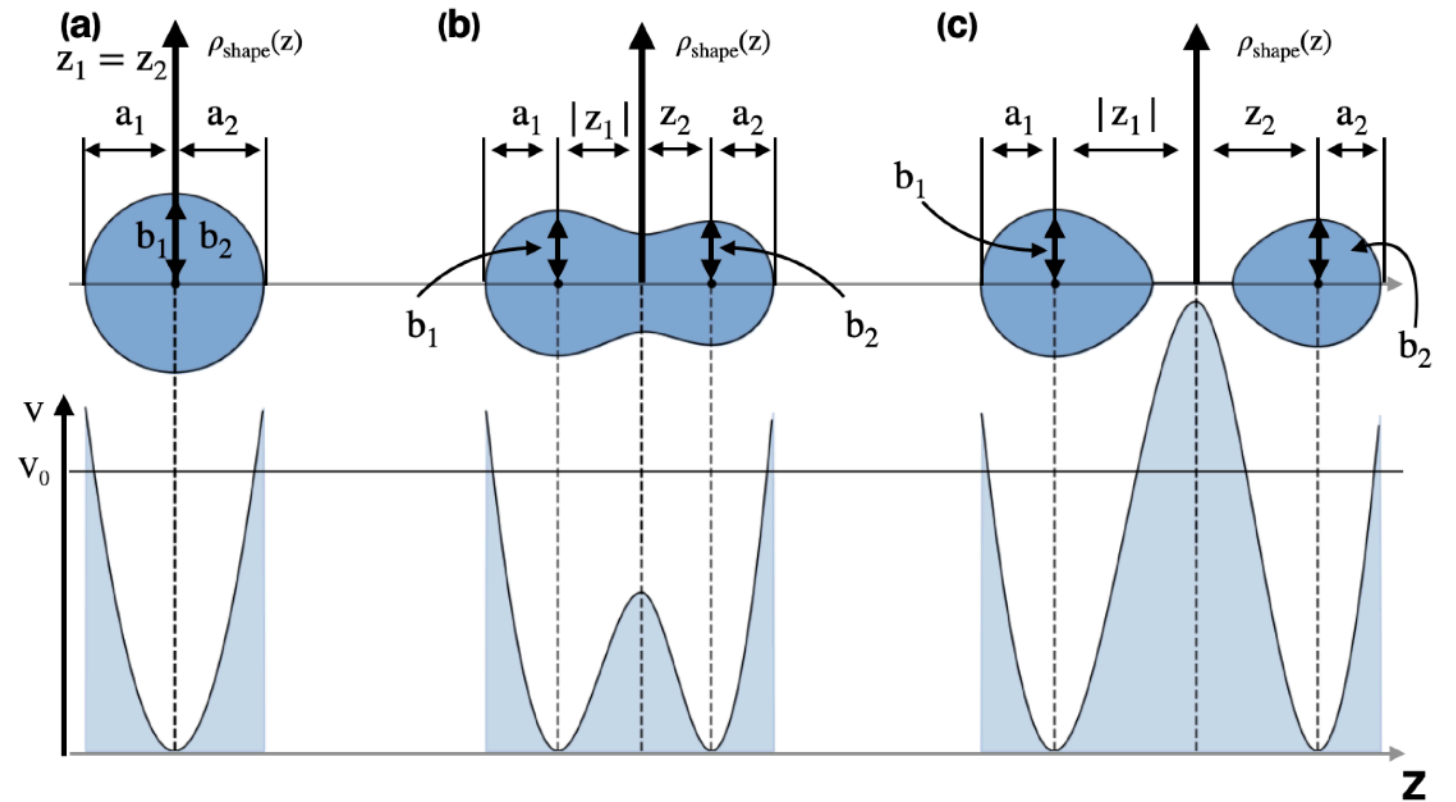


Degree of freedom - static degree of freedom

The parametrization originating from the **two center shell model (TCSM)**

$$\rho_{\text{shape}}^2(z) = \begin{cases} b_1^2 \left[1 - \frac{(z - z_1)^2}{a_1^2} \right], & z \leq z_1, \\ b_1^2 \left[1 - \frac{(z - z_1)^2}{a_1^2} (1 + c_1 z' + d_1 z'^2) \right] \frac{1}{(1 + g_1 z'^2)}, & z_1 < z < 0, \\ b_2^2 \left[1 - \frac{(z - z_2)^2}{a_2^2} (1 + c_2 z' + d_2 z'^2) \right] \frac{1}{(1 + g_2 z'^2)}, & 0 < z < z_2, \\ b_2^2 \left[1 - \frac{(z - z_2)^2}{a_2^2} \right], & z \geq z_2, \end{cases}$$

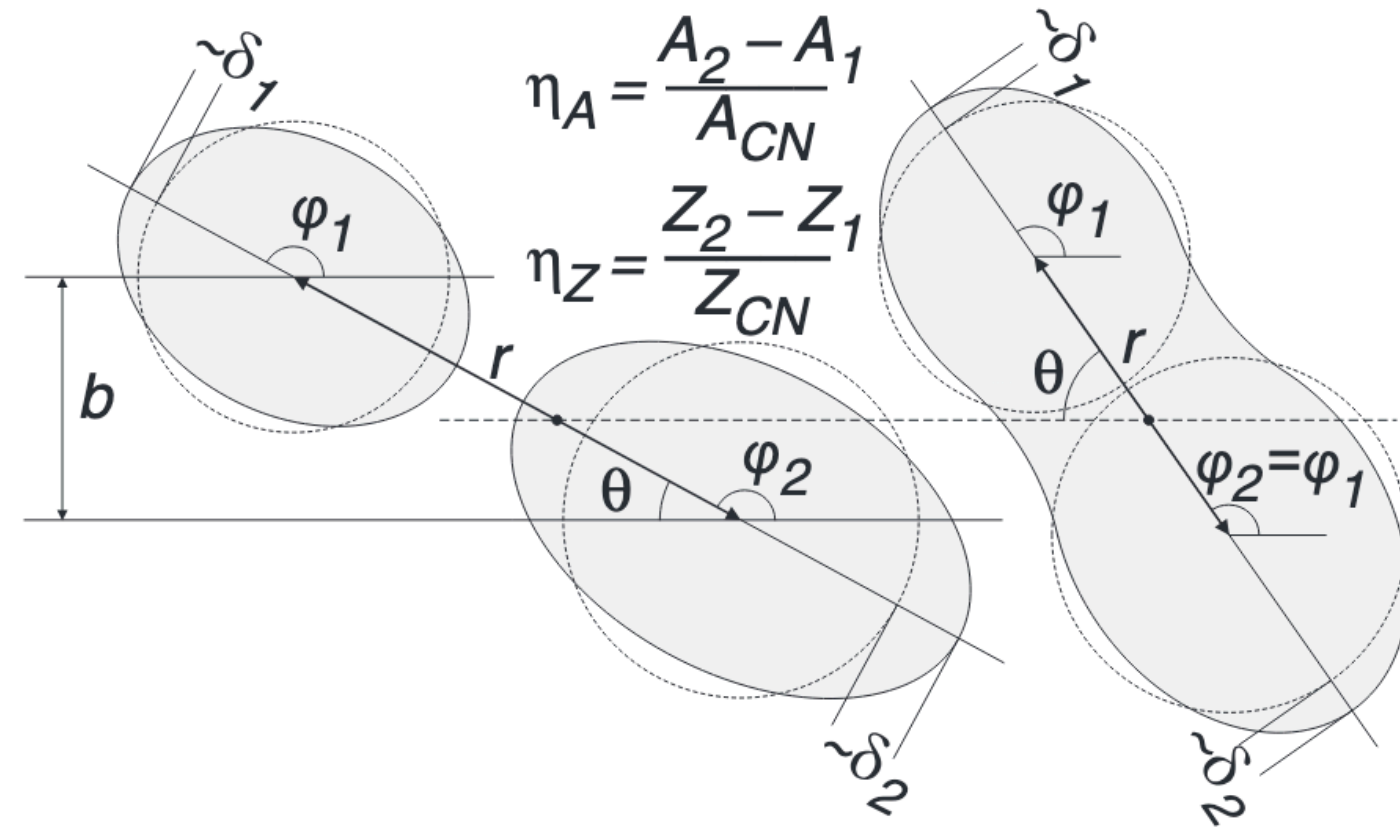
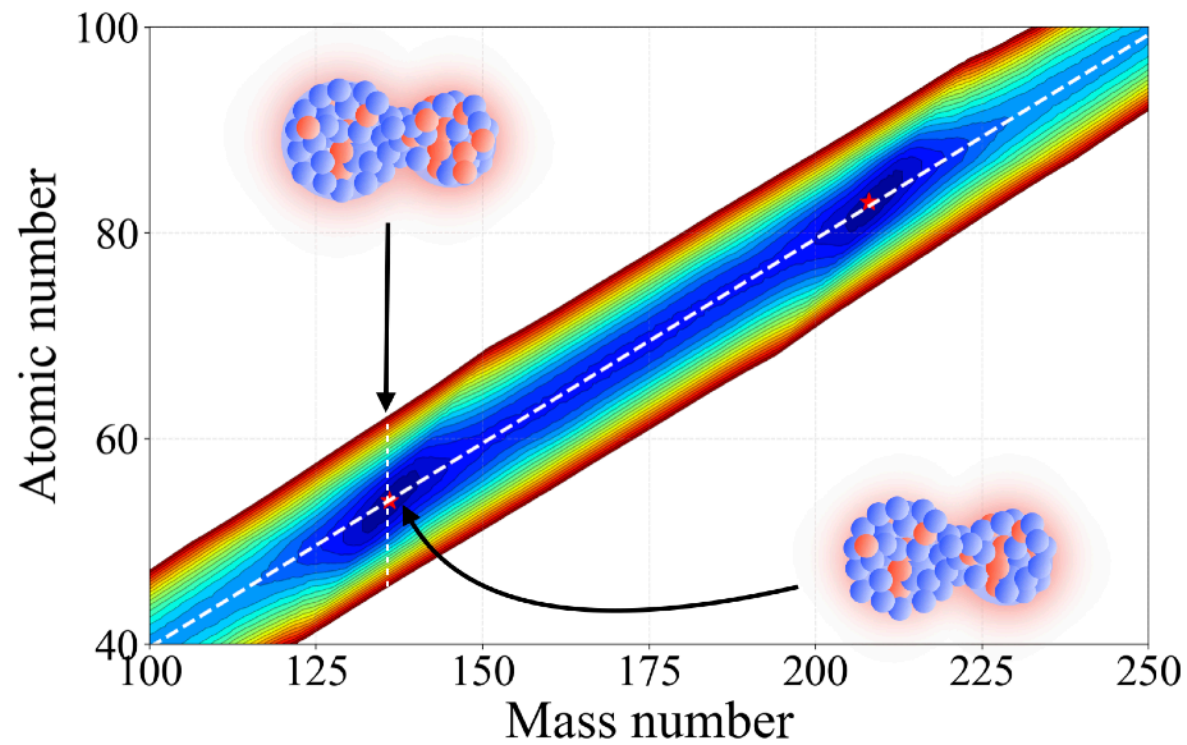
$$z' = \begin{cases} z - z_1, & z \leq 0, \\ z - z_2, & z > 0. \end{cases}$$



TCSM has some independent parameters

- $r = z_1 - z_2$: The distance between the oscillator centers,
- $\delta_{1(2)}$: The Ellipsoidal deformations, defined as $a_i/b_i - 1$
- η_A : The mass asymmetry, defined as $A_2 - A_1/(A_1 + A_2)$
- ε : The neck parameter. The smaller ε is, the thicker the neck

Degree of freedom - dynamical degree of freedom



- η_Z : the **charge asymmetry** between the left and right side
- θ : the **angle** between the symmetry axis and the beam direction
- $\varphi_{1(2)}$: the **rotation angle** of the left (right) nuclei

Degree of freedom

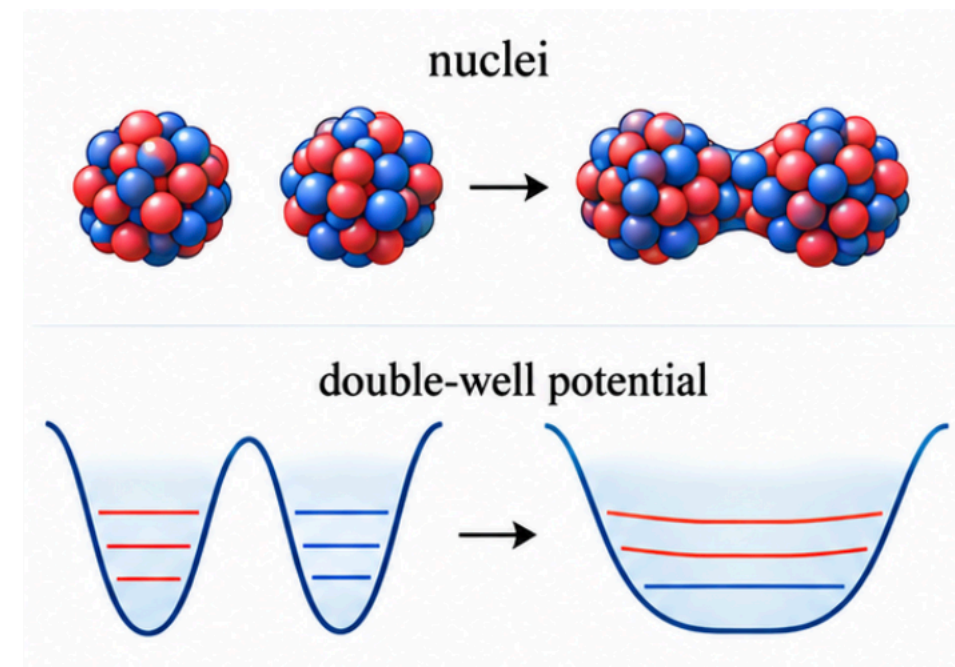
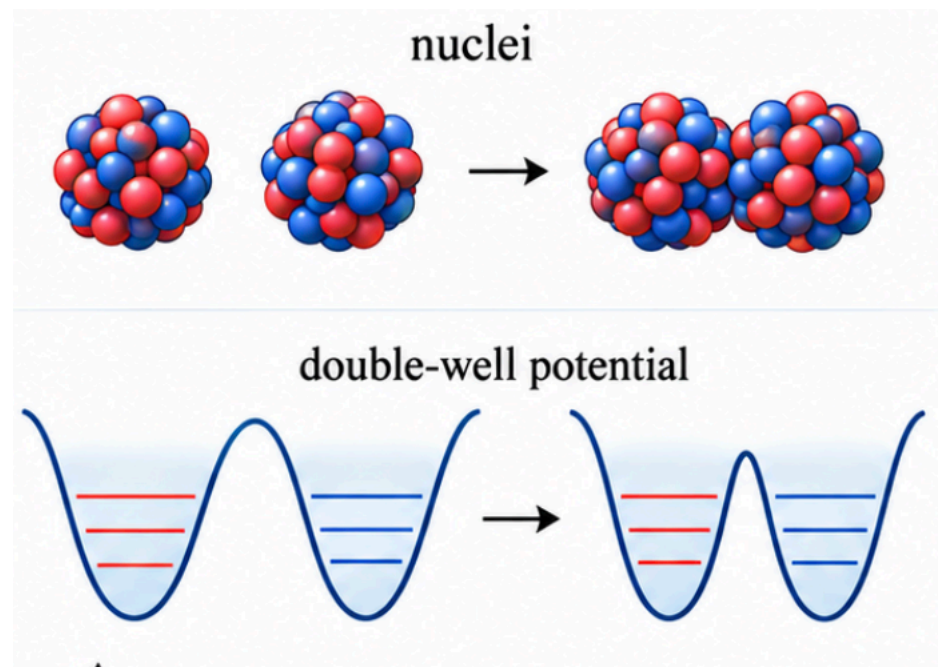


In this model, we consider **seven** degree of freedom, the collective coordinate include:

$$q_i = \{r, \delta_1 = \delta_2, \eta_A, \eta_Z, \theta, \varphi_1, \varphi_2\}$$

- r : the **distance** between two mass center
 - $\delta_{1(2)}$: the **deformation** of the left (right) side
 - η_A : the **mass asymmetry** between the left and right side
 - η_Z : the **charge asymmetry** between the left and right side
 - θ : the **angle** between the symmetry axis and the beam direction
 - $\varphi_{1(2)}$: the **rotation angle** of the left (right) nuclei
-

Potential energy surface

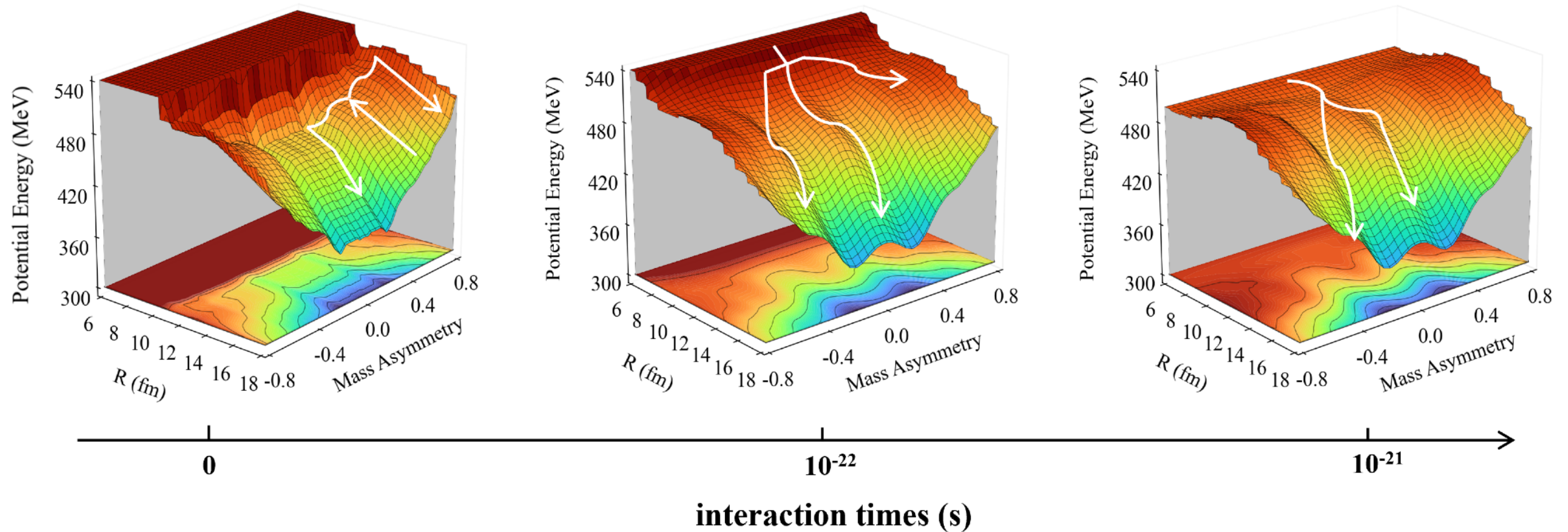


Then, we connect the diabatic and adiabatic potential energies with a time-dependent weighting function $f(t)$ as follows:

$$V(q, t) = V_{\text{diab}}(q)f(t) + V_{\text{adiab}}(q)[1 - f(t)] \quad f(t) = \exp\left(-\frac{t}{\tau_{\text{DA}}}\right), \quad \tau_{\text{DA}} = 10^{-22}\text{s}$$

Where q is a set of collective coordinates used

Potential energy surface



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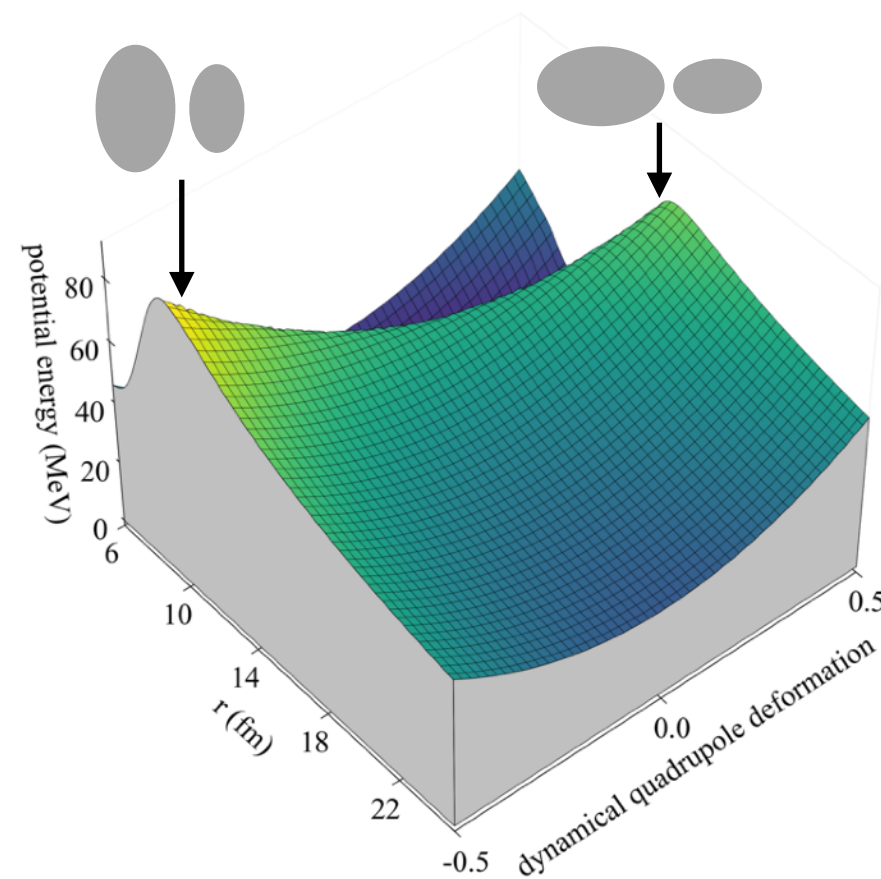
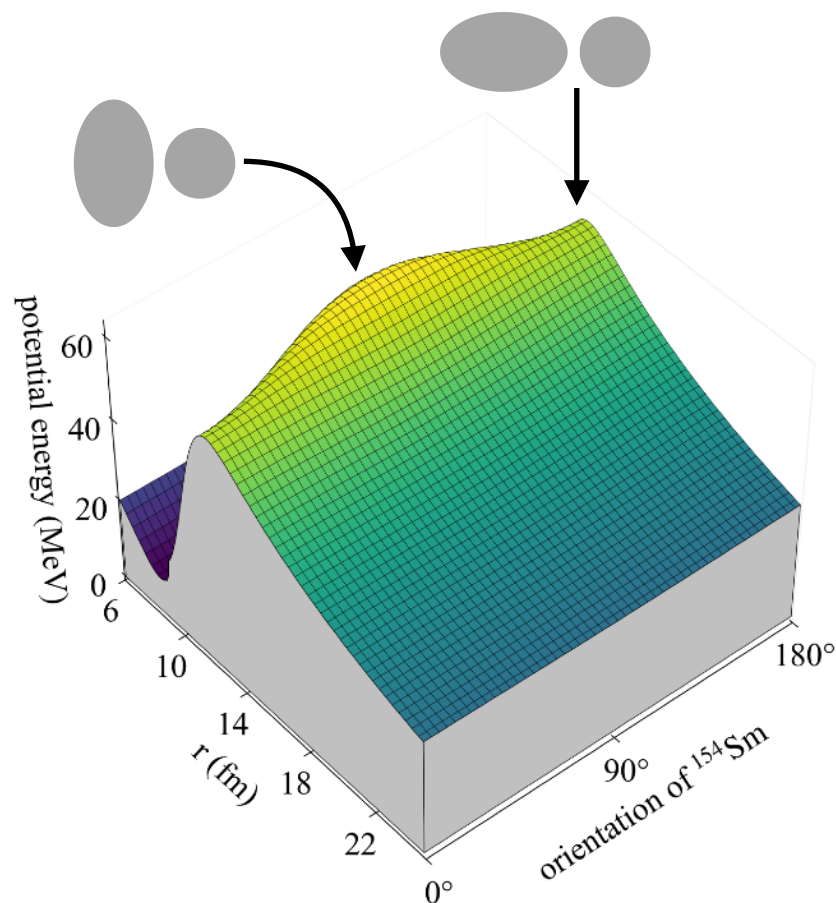
Potential energy surface - Diabatic potential energy

Redistribution of nucleons between separated nuclei changes their binding energies. The diabatic potential energy is defined as follows:

$$V_{\text{diab}}(A, Z; q) = \underbrace{V_{12}(A, Z; q)}_{\text{The interaction energy of the nuclei}} + \underbrace{M(A_1, Z_1; \delta_1)}_{\text{the masses of the reaction fragments}} + \underbrace{M(A_2, Z_2; \delta_2)}_{\text{the masses of the reaction fragments}} - \underbrace{M(A_T, Z_T; \delta_T^{\text{g.s.}})}_{\text{the masses of the entrance channel}} - \underbrace{M(A_P, Z_P; \delta_P^{\text{g.s.}})}_{\text{the masses of the entrance channel}}.$$

The interaction energy of the nuclei

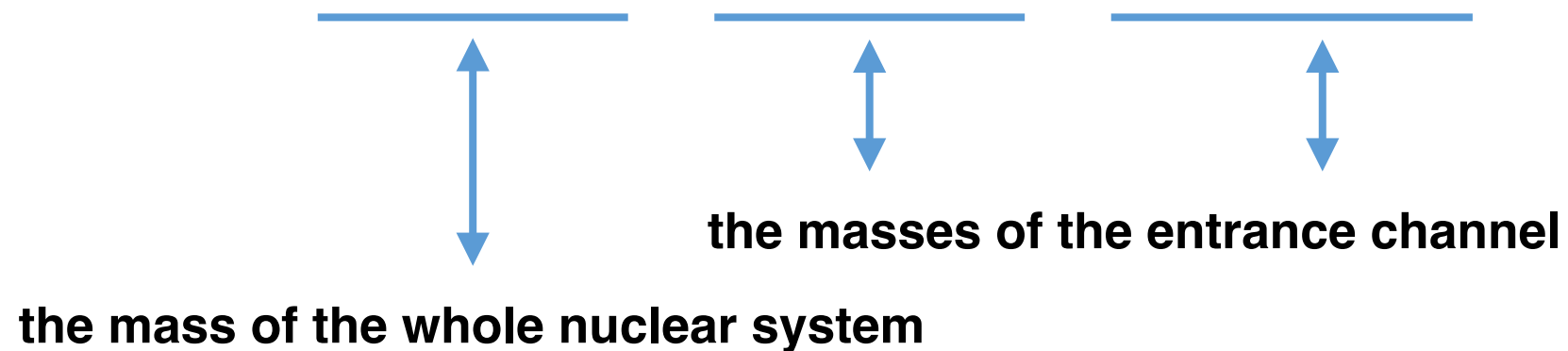
the masses of the entrance channel



Potential energy surface - Adiabatic potential energy

The adiabatic potential energy is defined as a difference between the mass of the whole nuclear system and the ground-state masses of the target and the projectile,

$$V_{\text{adiab}}(A, Z; q) = M(A, Z; q) - M(A_T, Z_T; \delta_T^{\text{g.s.}}) - M(A_P, Z_P; \delta_P^{\text{g.s.}}).$$



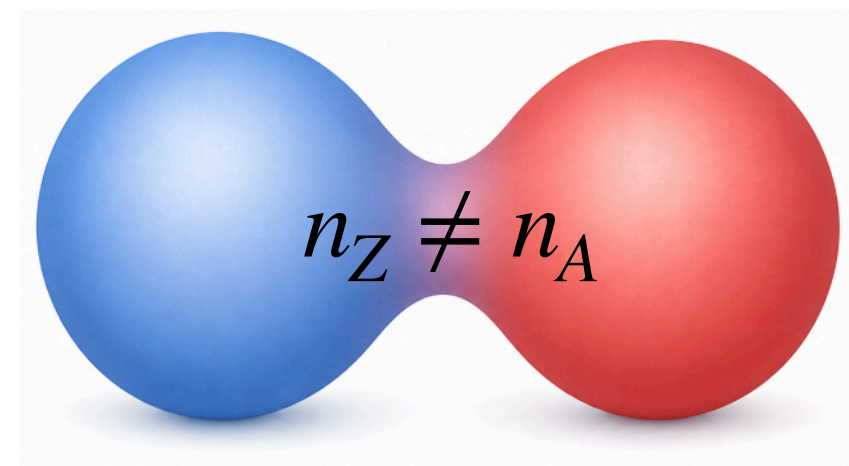
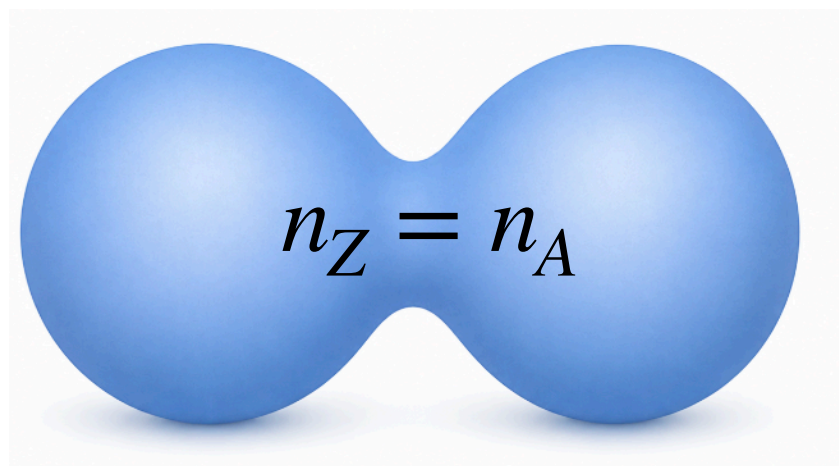
The standard macro-microscopic model is often used for calculation of the total mass:

$$M(A, Z; q) = M_{\text{mac}}(A, Z; q) + \delta E(A, Z; q).$$

- $M_{\text{mac}}(A, Z; q)$ is macroscopic liquid-drop mass
- $\delta E(A, Z; q)$ is the microscopic shell correction by Strutinsky shell-correction method

Potential energy surface - Adiabatic potential energy

$$\begin{aligned}
 M_{\text{mac}}(A, Z; \vec{q})|_{n_Z=n_A} = & m_p Z + m_n N \quad \text{----- mass excesses of } Z \text{ hydrogen atoms and } N \text{ neutrons} \\
 & -a_v(1 - k_v I^2)A + a_s(1 - k_s I^2)B_N(\vec{q})A^{2/3} \quad \text{----- volume energy, surface energy} \\
 & + \frac{3}{5} \frac{e^2 Z^2}{r_0 A^{1/3}} B_C(\vec{q}) - \frac{3}{4} \frac{e^2}{r_0} \left(\frac{9Z^4}{4\pi^2 A} \right)^{1/3} \quad \text{----- Coulomb energies, the Coulomb exchange correction} \\
 & + f(k_F r_p) \frac{Z^2}{A} - c_a(N - Z) \quad \text{----- the proton form-factor correction, the charge-asymmetry energy} \\
 & + a_0 \quad \text{----- the constant term} \\
 & + W \left(|I| + \begin{cases} 1/A, Z \text{ and } N \text{ equal and odd} \\ 0, \text{ otherwise} \end{cases} \right) \quad \text{----- the Wigner energy} \\
 & + \begin{cases} \Delta_p + \Delta_n - \delta_{np}, Z \text{ and } N \text{ odd} \\ \Delta_p, Z \text{ odd and } N \text{ even} \\ \Delta_n, Z \text{ even and } N \text{ odd} \\ 0, Z \text{ and } N \text{ even} \end{cases} \quad \text{----- the average pairing energy} \\
 & - a_{el} Z^{2.39} \quad \text{----- the energy of bound electrons}
 \end{aligned}$$



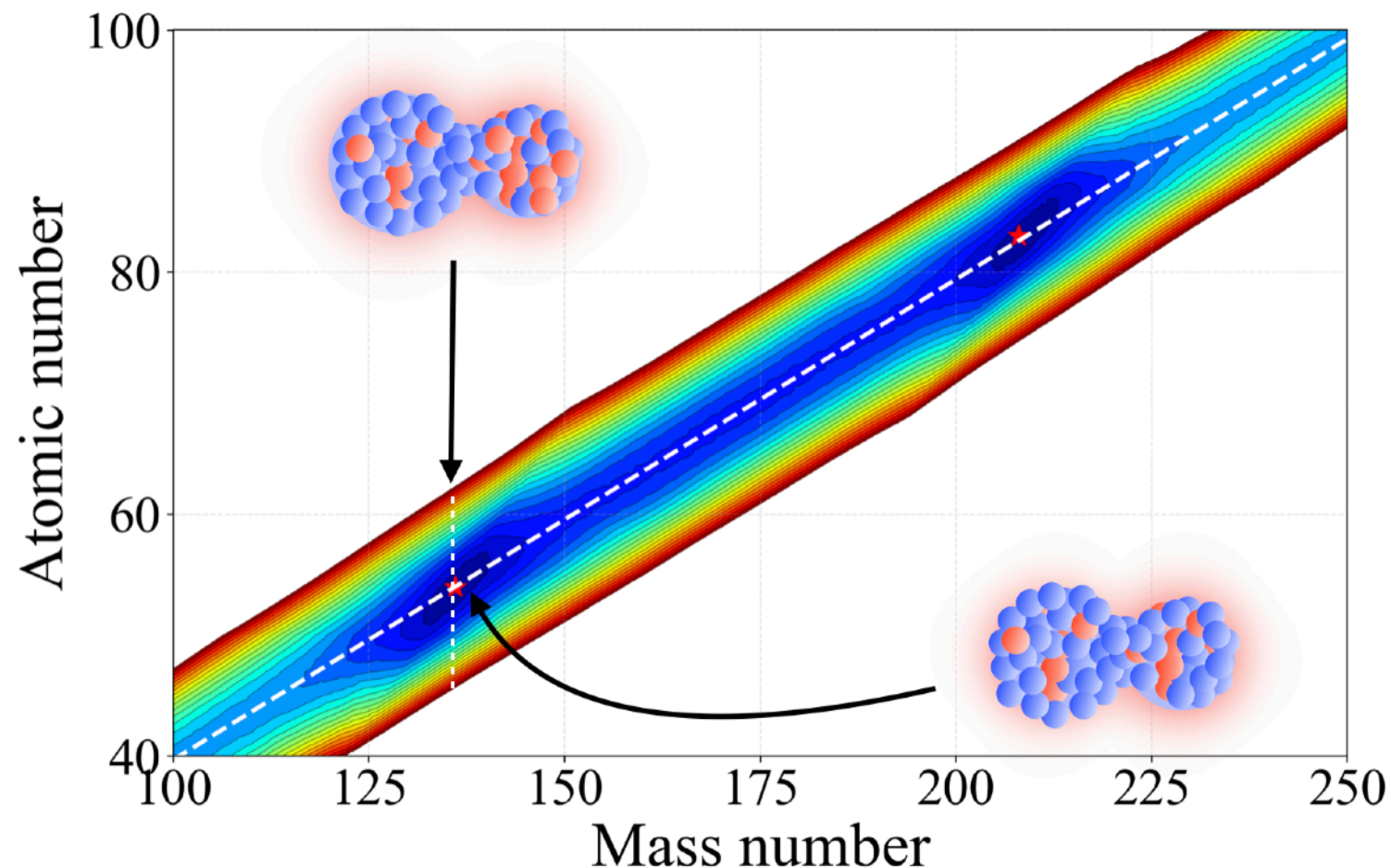
Potential energy surface - Adiabatic potential energy

In order to calculate the macroscopic mass for an arbitrary value of η_Z

$$M_{\text{mac}}(A, Z; q) \simeq M_{\text{mac}}(A, Z; \vec{q}) \big|_{\eta_Z = \langle \eta_Z \rangle} + C_{\eta_Z} \frac{(\eta_Z - \langle \eta_Z \rangle)^2}{2},$$

where $\langle \eta_Z \rangle$ is the **charge asymmetry** providing the minimum of the potential energy and C_{η_Z} is the **stiffness coefficient**.

$$M_{\text{mac}}(A, Z; q) \approx M_{\text{mac}}(A, Z; \vec{q}) \big|_{n_Z = n_A} - C_{\eta_Z} \frac{(\eta_A - \langle \eta_Z \rangle)^2}{2} + C_{\eta_Z} \frac{(\eta_Z - \langle \eta_Z \rangle)^2}{2},$$



Potential energy surface - The Strutinsky method

The Hamiltonian of the TCSM in cylindrical coordinates can be written as

$$H = -\frac{\hbar^2 \nabla^2}{2m} + V(\rho, z) + V_{LS}(r, l, s) + V_{L^2}(r, l)$$

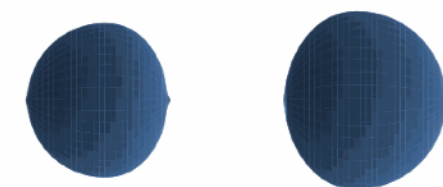
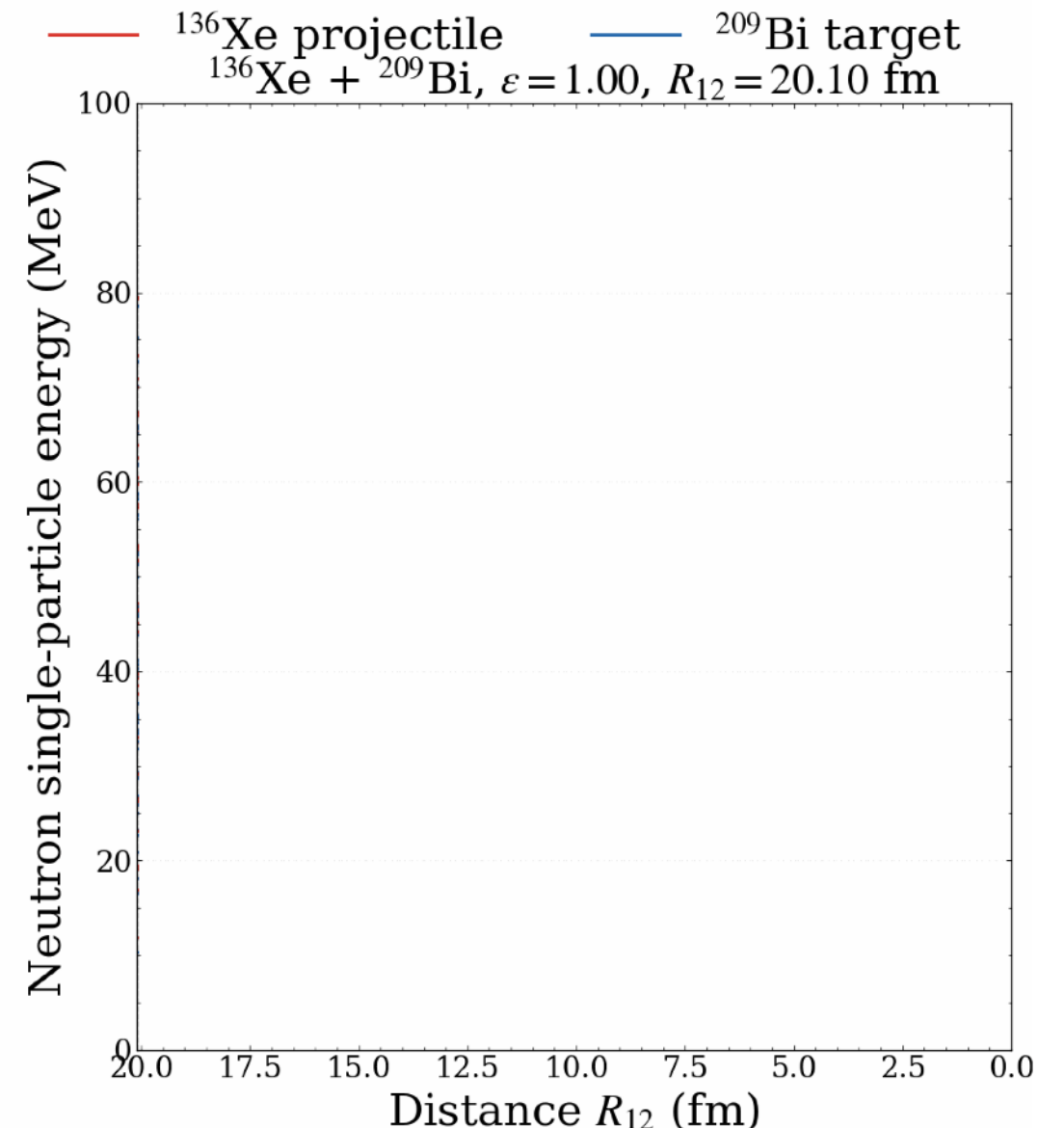
Two-center potential

$$V_i(\rho, z) \simeq \frac{1}{2}m_0\omega_{zi}^2(z - z_i)^2 + \frac{1}{2}m_0\omega_{\rho i}^2\rho^2, \quad i = 1, 2.$$

For a given nuclear shape, the single-particle states are obtained from

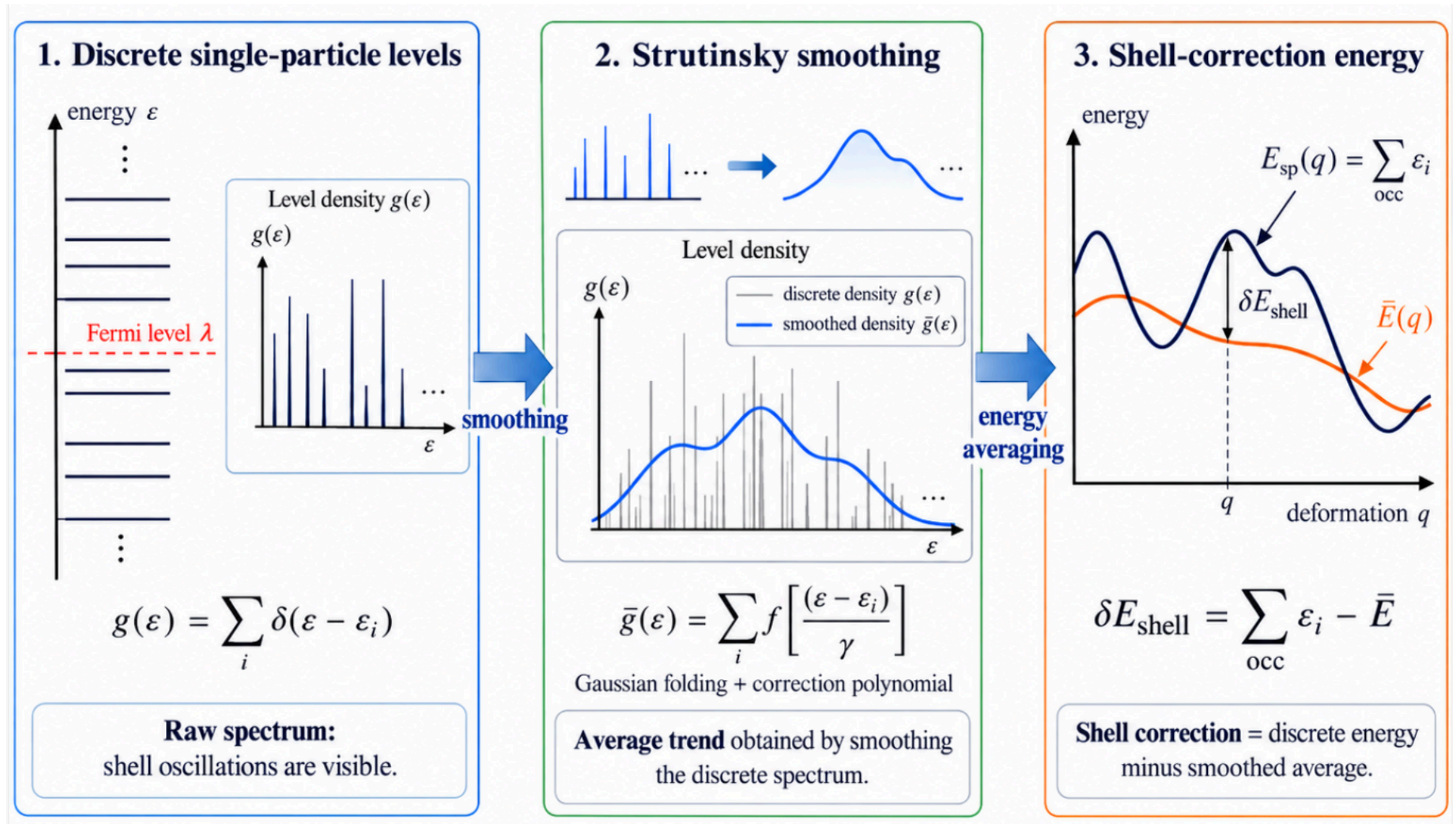
$$\hat{H}\Phi_\alpha = \epsilon_\alpha\Phi_\alpha, \quad \Phi(\rho, z, \phi) = f(z)g(\rho)h(\phi).$$

so the basis states can be written as $|n_z n_\rho m\rangle$.

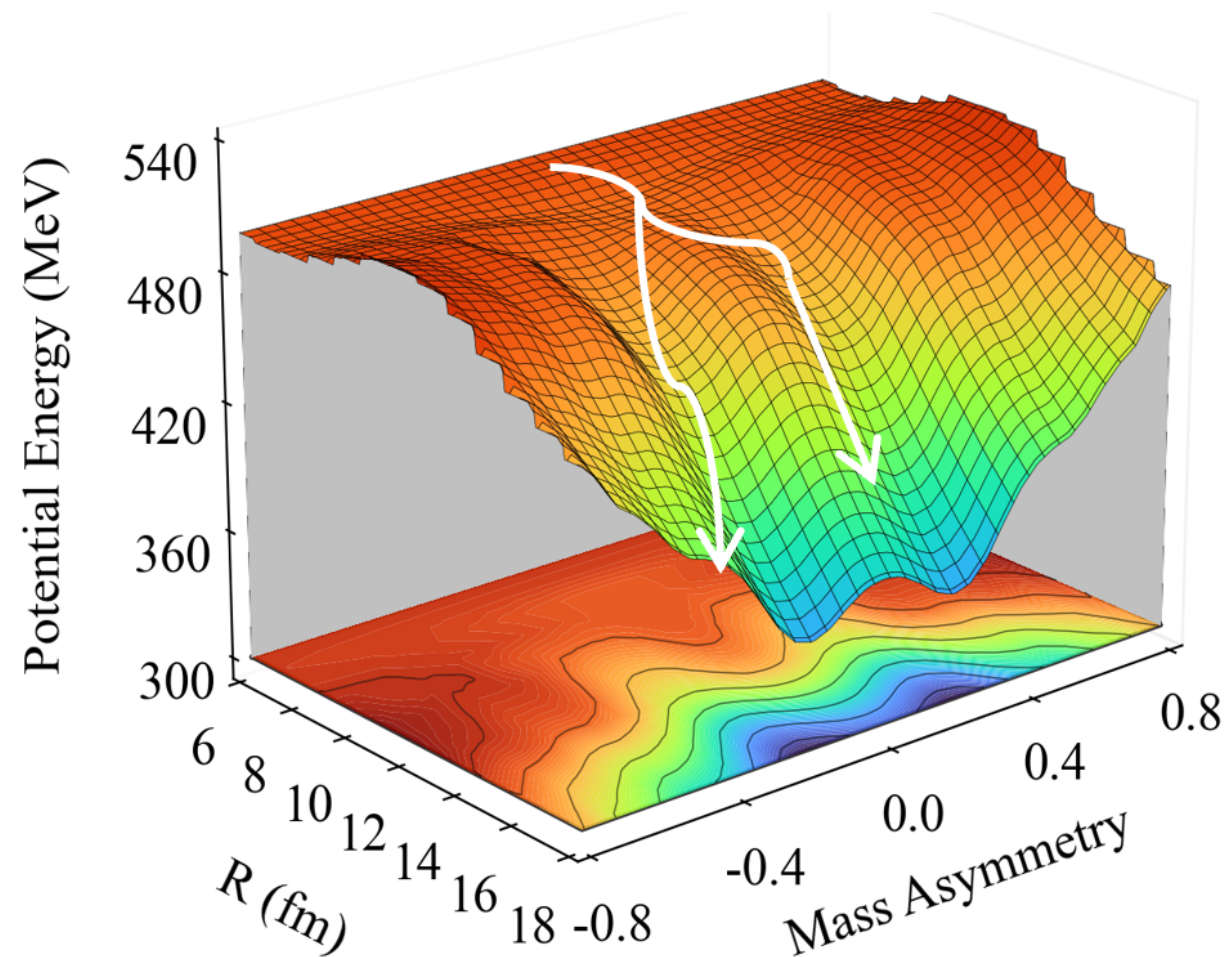


Potential energy surface - The Strutinsky method

The shell correction reflects the deviation of single-particle energy levels from a uniform distribution.



Equations of motion and nucleon transfer



Langevin equation

$$\dot{q}_i = \mu_{ij} p_j, \quad \dot{p}_i = F_i^{\text{driving}} + F_i^{\text{friction}} + F_i^{\text{random}}$$

$$q_i = \{r, \delta_1 = \delta_2, \theta, \varphi_1, \varphi_2\}$$

$$p_i = \{p_r, p_{\delta_1} = p_{\delta_2}, L, l_1, l_2\}$$

- Describing single event shape evolution

Master equation

$$\frac{\partial P(\mathbf{S}, t)}{\partial t} = \sum_{\mathbf{S}' \neq \mathbf{S}} W(\mathbf{S}', \mathbf{S}) P(\mathbf{S}', t) - W(\mathbf{S}, \mathbf{S}') P(\mathbf{S}, t)$$

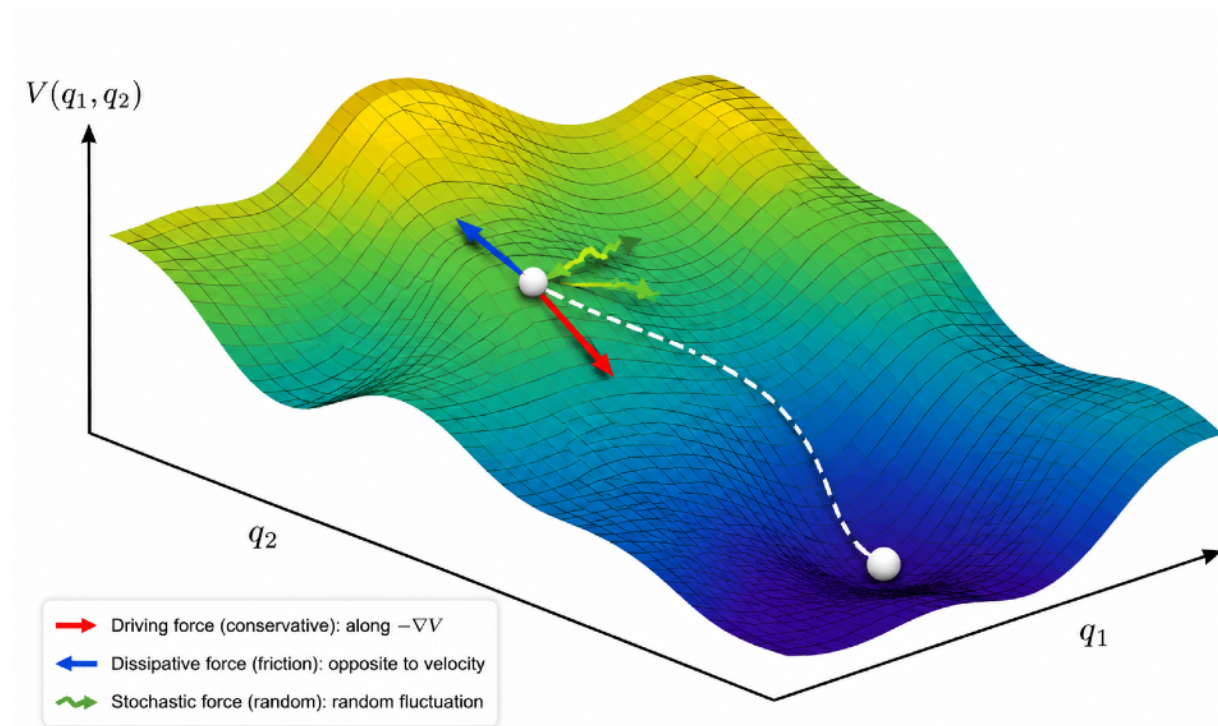
$$\mathbf{S} = \{N, Z\}$$

- Describing nucleon transfer evolution

Equations of motion and nucleon transfer

Modeling the time evolution of the nuclear system, we can use the Langevin equation

$$\dot{q}_i = \mu_{ij} p_j, \quad \dot{p}_i = F_i^{\text{driving}} + F_i^{\text{friction}} + F_i^{\text{random}}$$



$$\begin{aligned} \frac{dp_r}{dt} &= -\frac{\partial V}{\partial r} - \sum_{j,k} \frac{p_j p_k}{2} \frac{\partial \mu_{jk}}{\partial r} + \frac{\hbar^2 l^2}{2\mathfrak{I}_{12}^2} \frac{\partial \mathfrak{I}_{12}}{\partial r} + \sum_{i=1,2} \frac{\hbar^2 l_i^2}{2\mathfrak{I}_i^2} \frac{\partial \mathfrak{I}_i}{\partial r} - \sum_{j,k} \gamma_{rj} \mu_{jk} p_k + \sum_j g_{rj} \Gamma_j \\ \frac{dp_{\delta_1}}{dt} &= -\frac{\partial V}{\partial \delta_1} - \sum_{j,k} \frac{p_j p_k}{2} \frac{\partial \mu_{jk}}{\partial \delta_1} + \frac{\hbar^2 l^2}{2\mathfrak{I}_{12}^2} \frac{\partial \mathfrak{I}_{12}}{\partial \delta_1} + \frac{\hbar^2 l_1^2}{2\mathfrak{I}_1^2} \frac{\partial \mathfrak{I}_1}{\partial \delta_1} - \sum_{j,k} \gamma_{\delta_1 j} \mu_{jk} p_k + \sum_j g_{\delta_1 j} \Gamma_j \\ \frac{dp_{\delta_2}}{dt} &= -\frac{\partial V}{\partial \delta_2} - \sum_{j,k} \frac{p_j p_k}{2} \frac{\partial \mu_{jk}}{\partial \delta_2} + \frac{\hbar^2 l^2}{2\mathfrak{I}_{12}^2} \frac{\partial \mathfrak{I}_{12}}{\partial \delta_2} + \frac{\hbar^2 l_2^2}{2\mathfrak{I}_2^2} \frac{\partial \mathfrak{I}_2}{\partial \delta_2} - \sum_{j,k} \gamma_{\delta_2 j} \mu_{jk} p_k + \sum_j g_{\delta_2 j} \Gamma_j \\ \frac{dl}{dt} \hbar &= -\frac{\partial V}{\partial \theta} - \gamma_{\text{tang}} \left(\frac{\hbar l}{\mathfrak{I}_{12}} R_{\text{c.m.}} - \frac{\hbar l_1}{\mathfrak{I}_1} R_{\text{c.m.}}^{(1)} - \frac{\hbar l_2}{\mathfrak{I}_2} R_{\text{c.m.}}^{(2)} \right) R_{\text{c.m.}} + R_{\text{c.m.}} \sqrt{\gamma_{\text{tang}} T} \Gamma_{\text{tang}} \\ \frac{dl_1}{dt} \hbar &= -\frac{\partial V}{\partial \varphi_1} + \gamma_{\text{tang}} \left(\frac{\hbar l}{\mathfrak{I}_{12}} R_{\text{c.m.}} - \frac{\hbar l_1}{\mathfrak{I}_1} R_{\text{c.m.}}^{(1)} - \frac{\hbar l_2}{\mathfrak{I}_2} R_{\text{c.m.}}^{(2)} \right) R_{\text{c.m.}}^{(1)} - R_{\text{c.m.}}^{(1)} \sqrt{\gamma_{\text{tang}} T} \Gamma_{\text{tang}} \\ \frac{dl_2}{dt} \hbar &= -\frac{\partial V}{\partial \varphi_2} + \gamma_{\text{tang}} \left(\frac{\hbar l}{\mathfrak{I}_{12}} R_{\text{c.m.}} - \frac{\hbar l_1}{\mathfrak{I}_1} R_{\text{c.m.}}^{(1)} - \frac{\hbar l_2}{\mathfrak{I}_2} R_{\text{c.m.}}^{(2)} \right) R_{\text{c.m.}}^{(2)} - R_{\text{c.m.}}^{(2)} \sqrt{\gamma_{\text{tang}} T} \Gamma_{\text{tang}} \end{aligned}$$

Equations of motion and nucleon transfer

A master equation is a phenomenological set of first-order **differential equations** describing the time evolution of (usually) the **probability** of a system to occupy each one of a discrete **set** of **states** with regard to a continuous time variable t .

$$\frac{\partial P(S, t)}{\partial t} = \sum_{S' \neq S} W(S', S)P(S', t) - W(S, S')P(S, t)$$

The transition probability from the state to another state

**Macroscopic transition
probabilities**



**Microscopic transition
probabilities**



$$W^{\pm} = \lambda_0 \cdot D(q, S) \cdot P_{tr}(q, S \rightarrow S \pm 1)$$



Diffusion constant



Probability of a nucleon transfer

The transition probability from the state to another state

Macroscopic transition probabilities

Microscopic transition probabilities



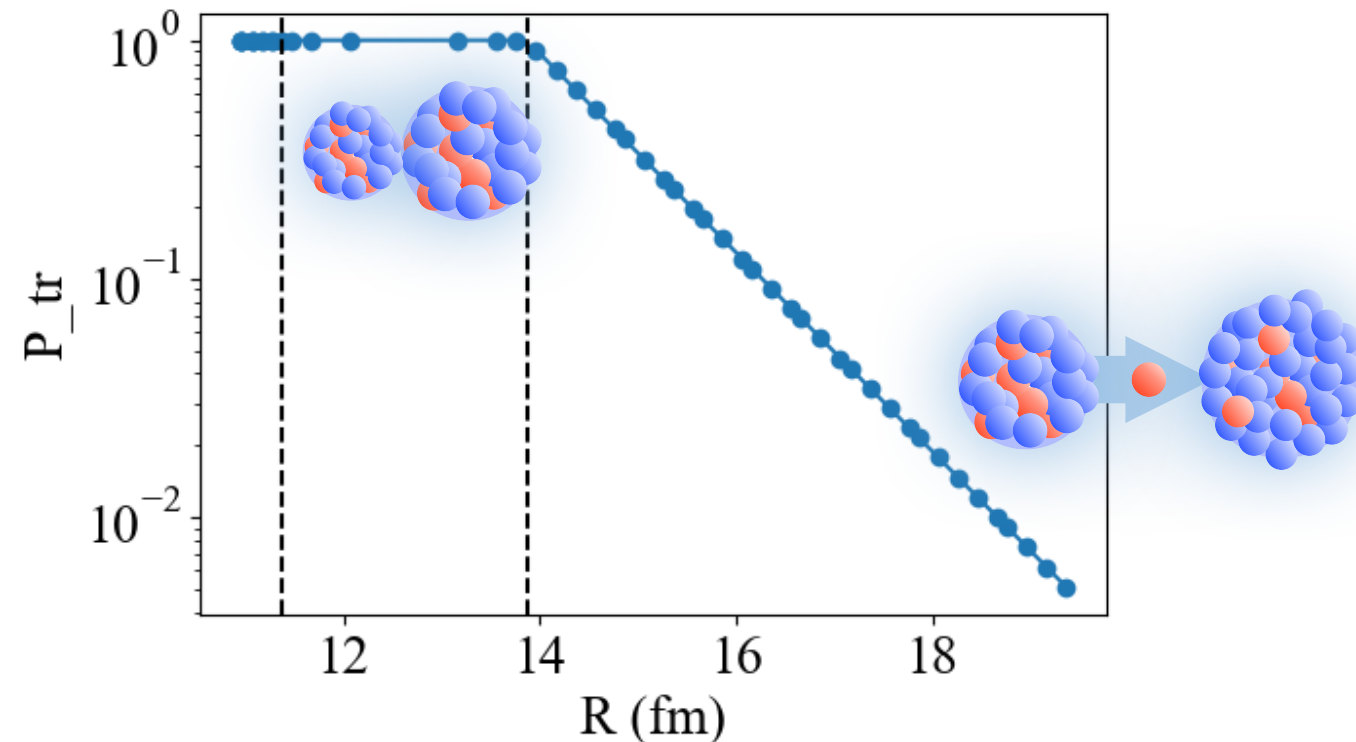
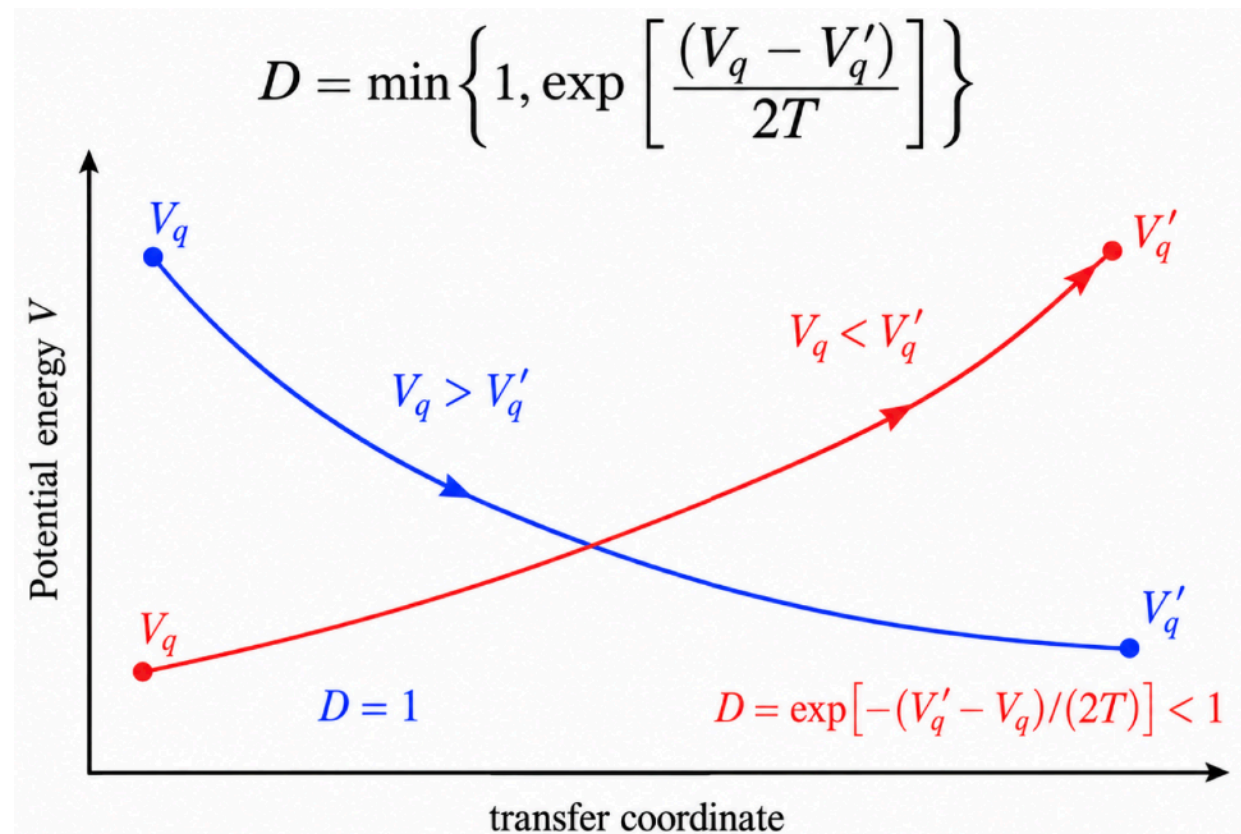
$$W^{\pm} = \lambda_0 \cdot D(q, S) \cdot P_{tr}(q, S \rightarrow S \pm 1)$$



Diffusion constant



Probability of a nucleon transfer

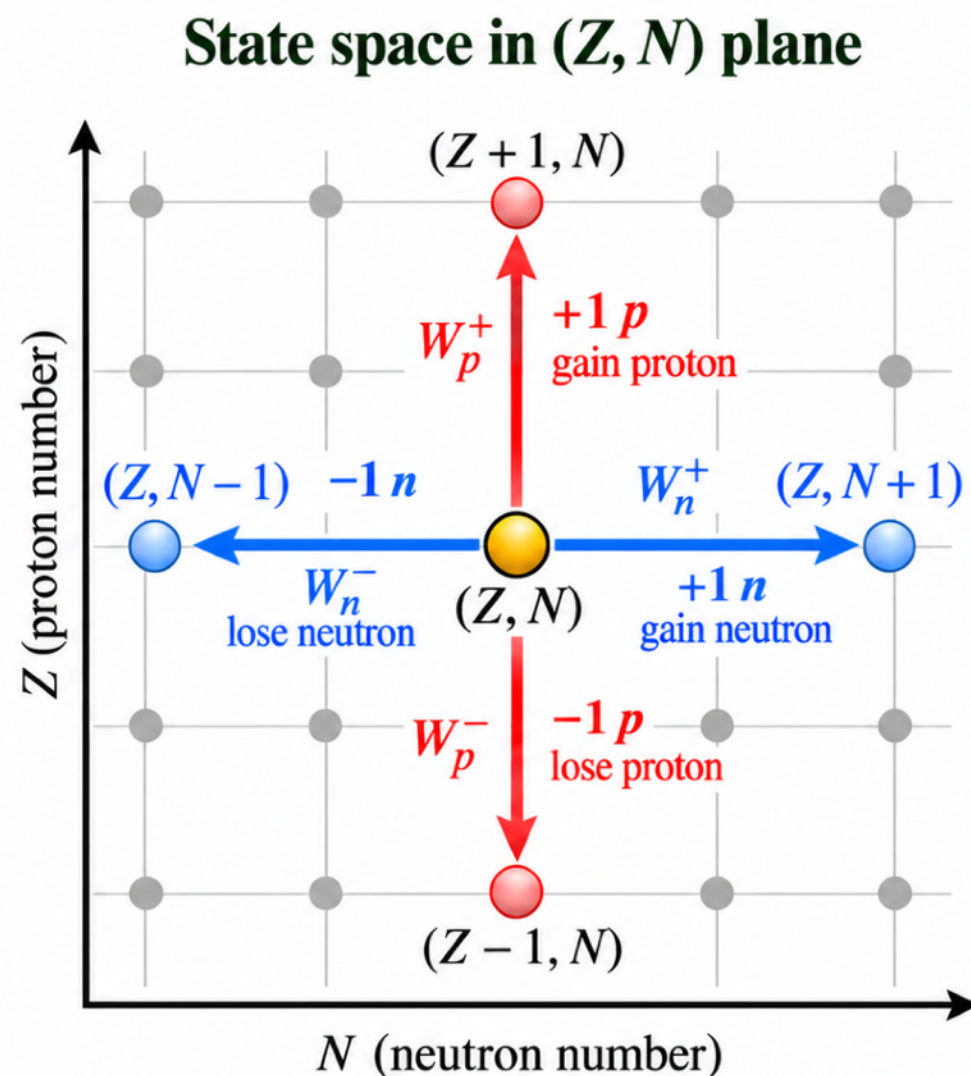


$$P_{tr}(R, \beta, A \rightarrow A \pm 1) = \exp(-2k[r_c - r_0])$$

Equations of motion and nucleon transfer

A master equation is a phenomenological set of first-order **differential equations** describing the time evolution of (usually) the **probability** of a system to occupy each one of a discrete **set of states** with regard to a continuous time variable t .

$$\frac{\partial P(S, t)}{\partial t} = \sum_{S' \neq S} W(S', S)P(S', t) - W(S, S')P(S, t)$$

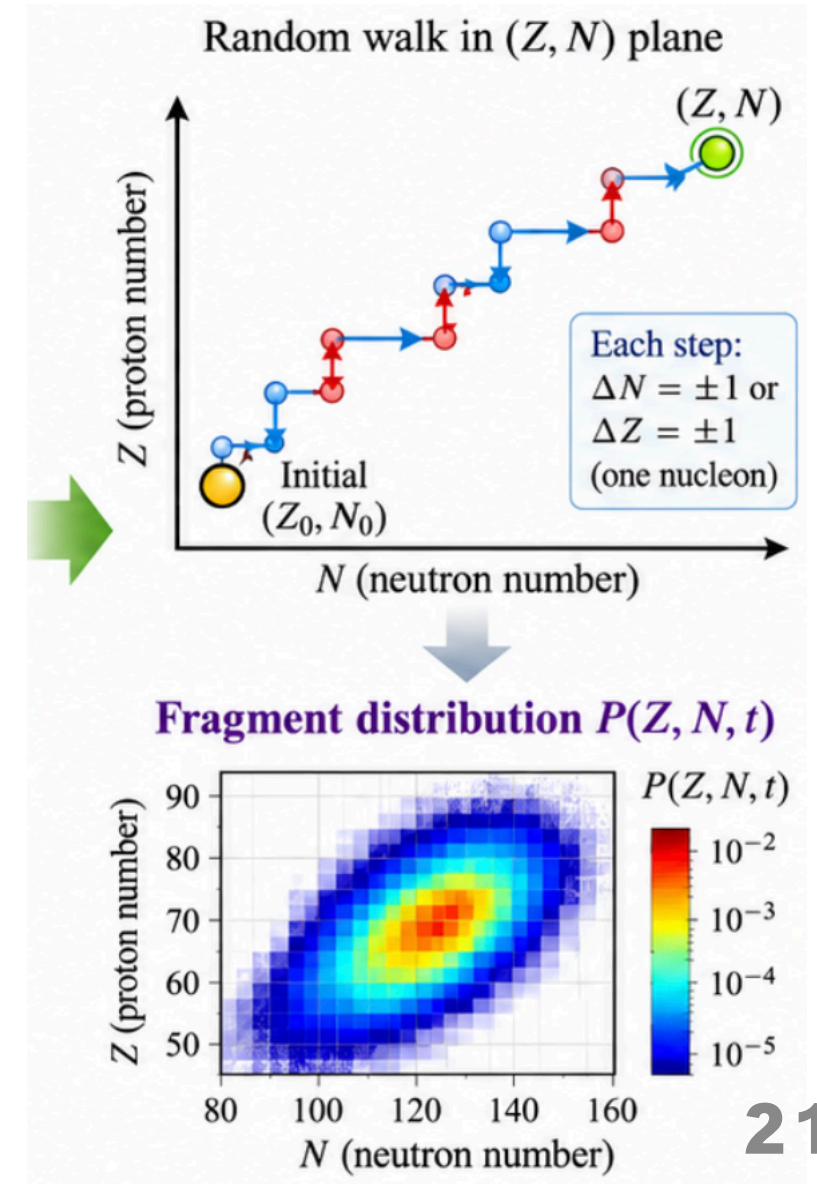


KMC step

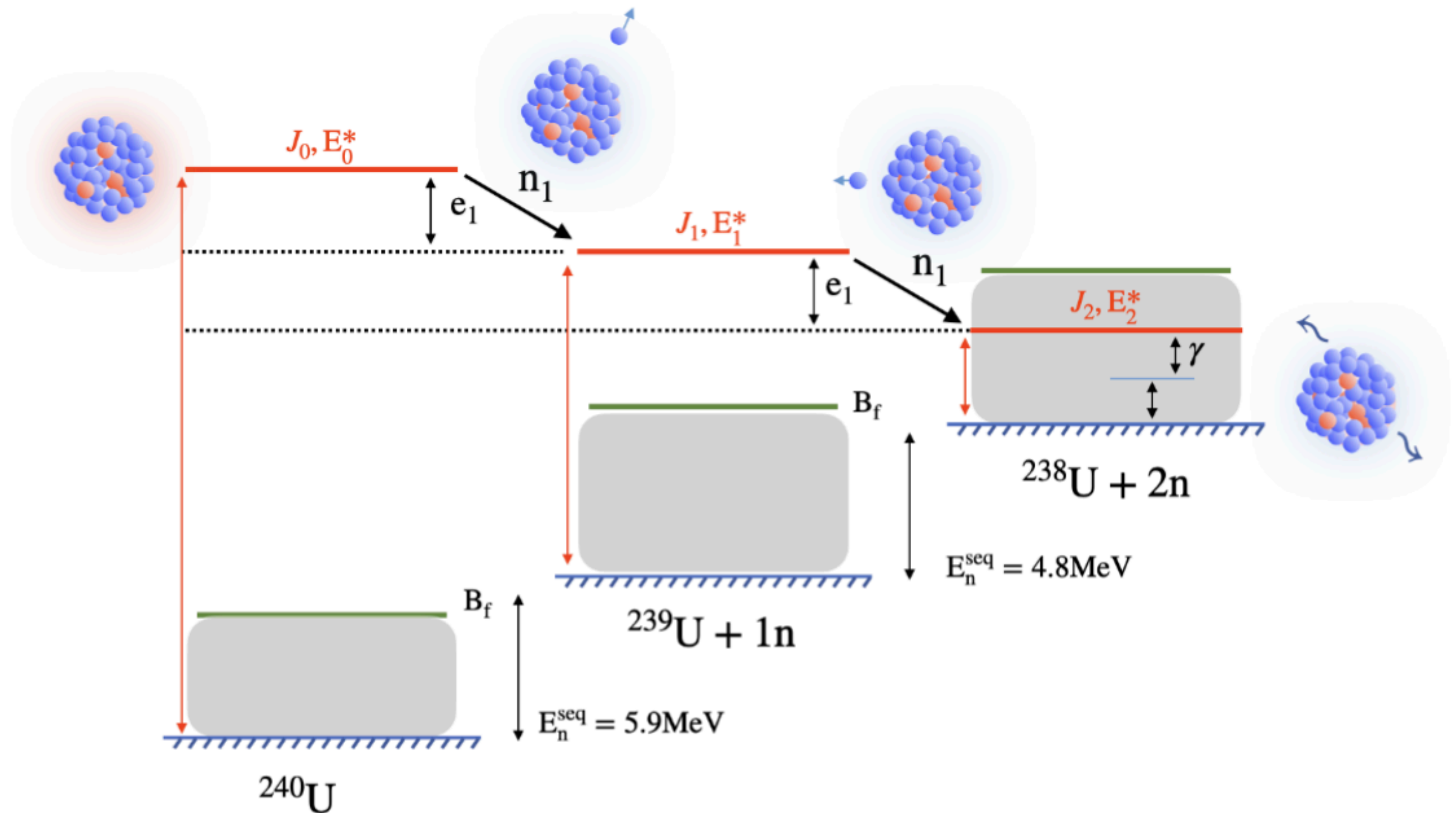
- 1 Calculate rates:
 $W_n^+, W_n^-, W_p^+, W_p^-$
- 2 Draw waiting time Δt
- 3 Select one of four channels
(probability proportional to W)
- 4 Update (Z, N)



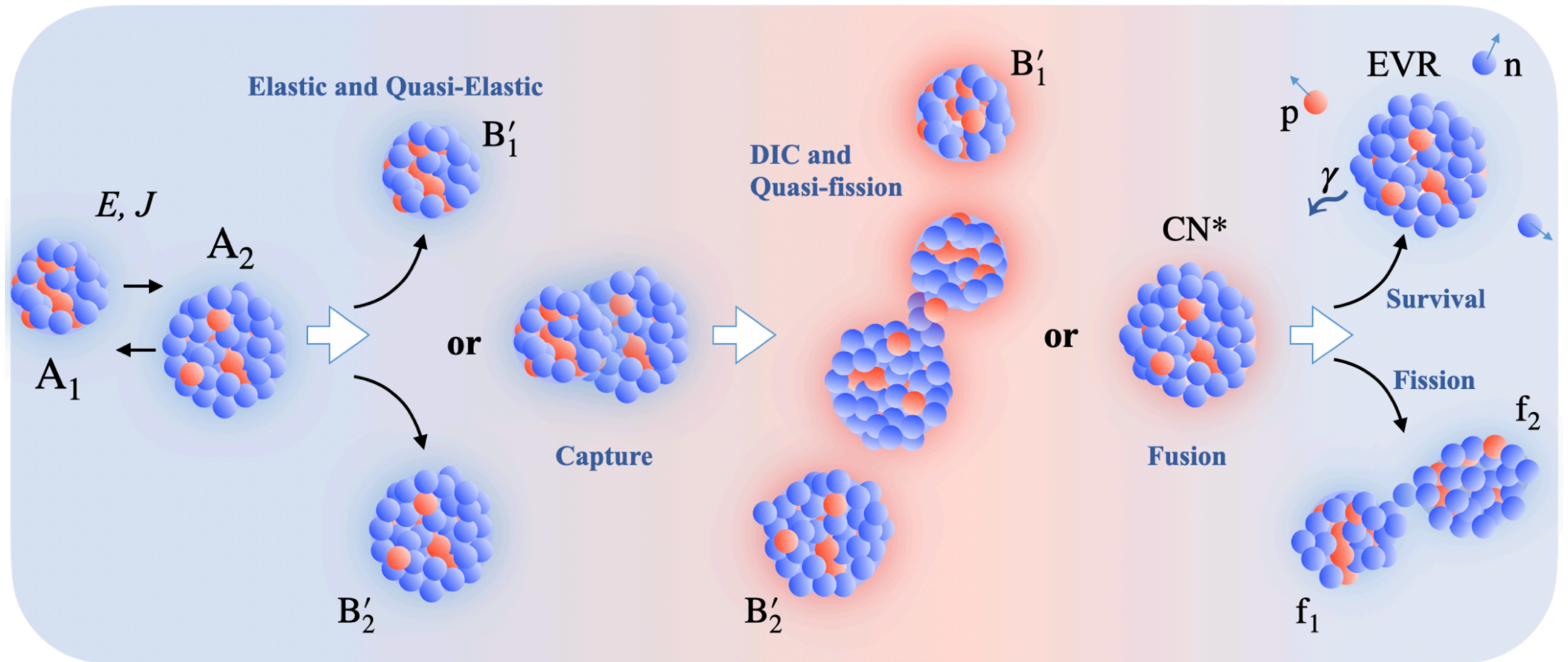
Random sampling
(stochastic choice)



Modeling nucleus-nucleus collisions

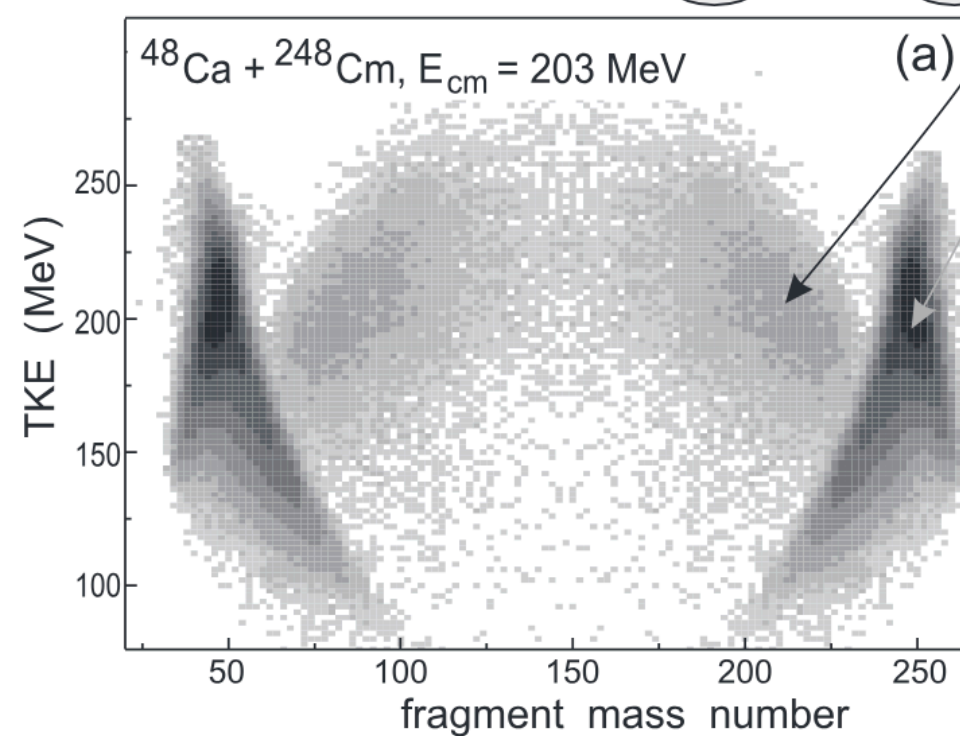
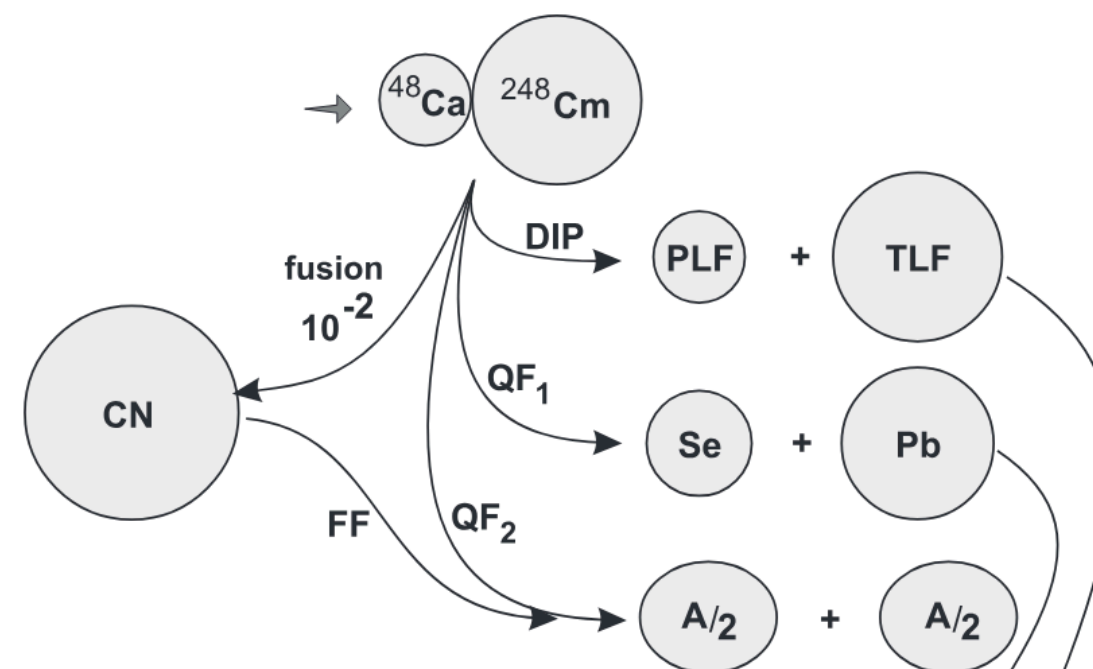
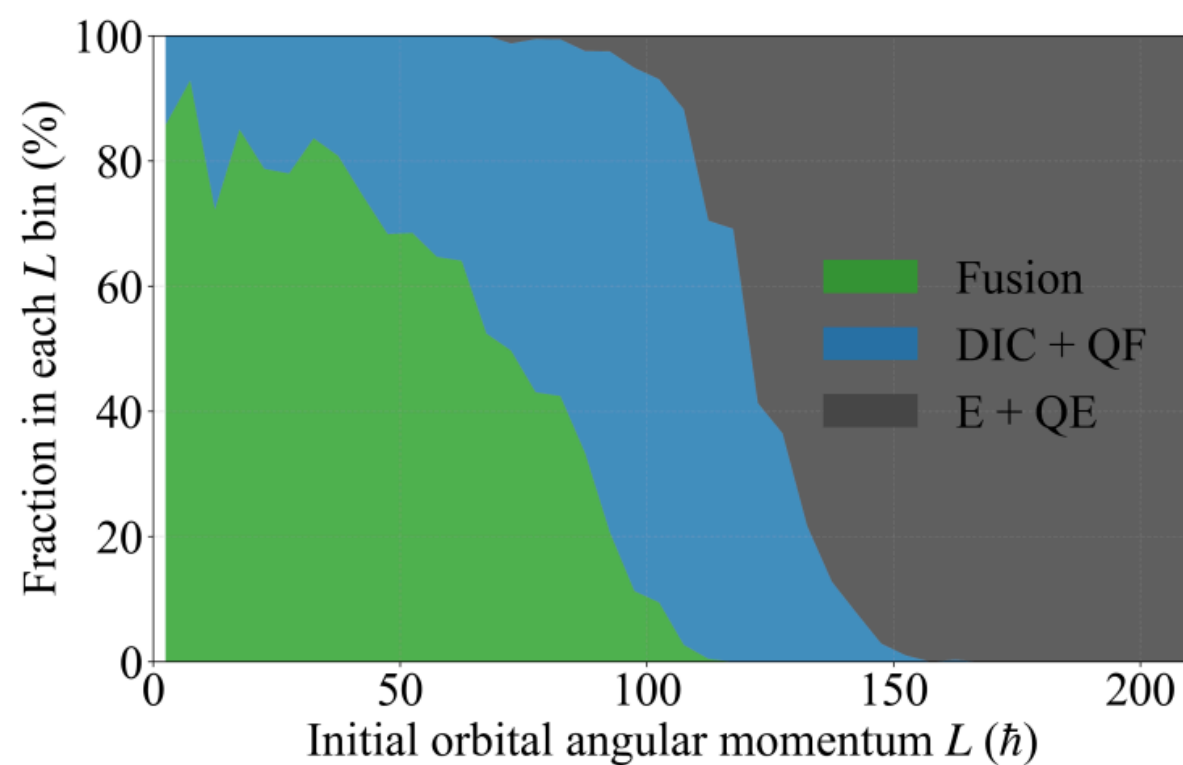
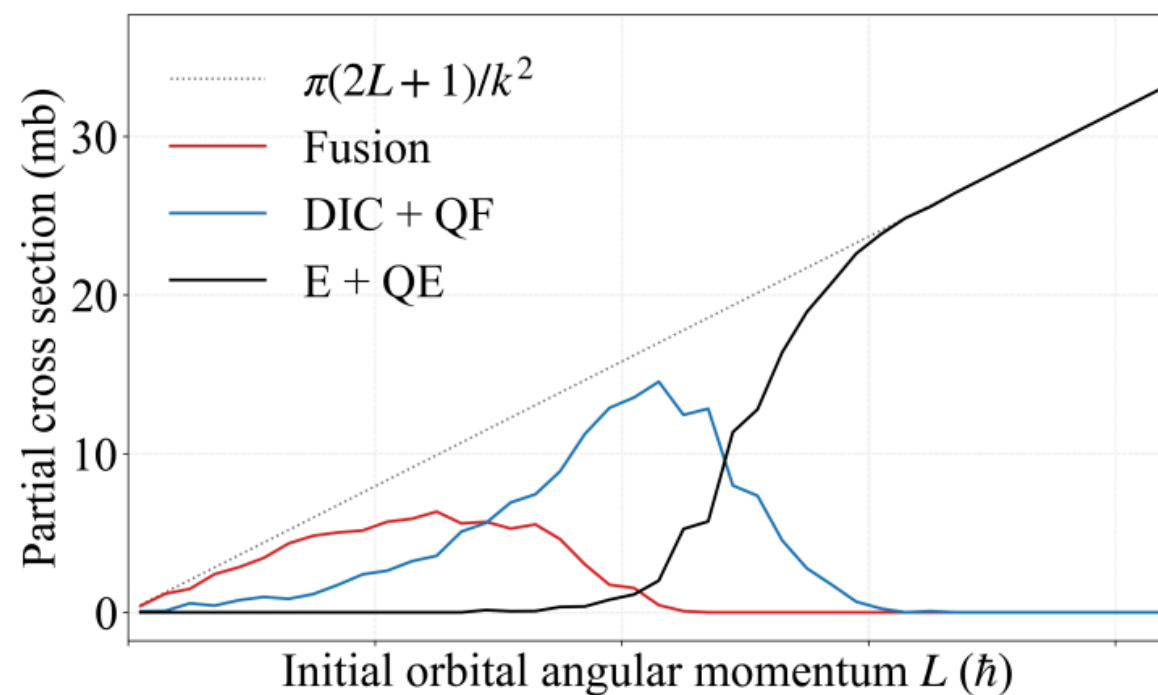


Modeling nucleus-nucleus collisions



$$\sigma(Z_f, N_f) = \frac{\pi \hbar^2}{2\mu E_{\text{c.m.}}} \sum_{J=0}^{J_{\text{max}}} (2J+1) \sum_{Z_1, N_1} P(Z_1, N_1; J^*, E^*) W(Z_1, N_1 \rightarrow Z_f, N_f; J^*, E^*),$$

Modeling nucleus-nucleus collisions

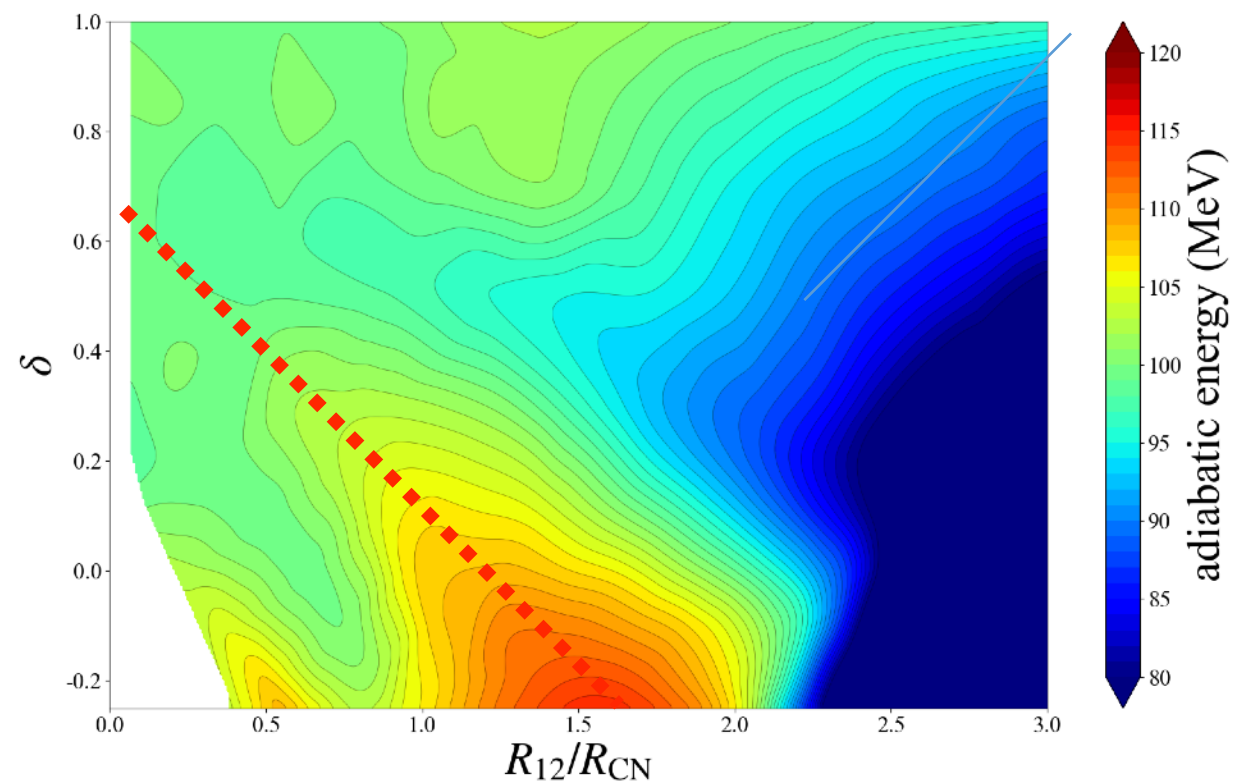


Modeling nucleus-nucleus collisions

Capture and non-Capture:

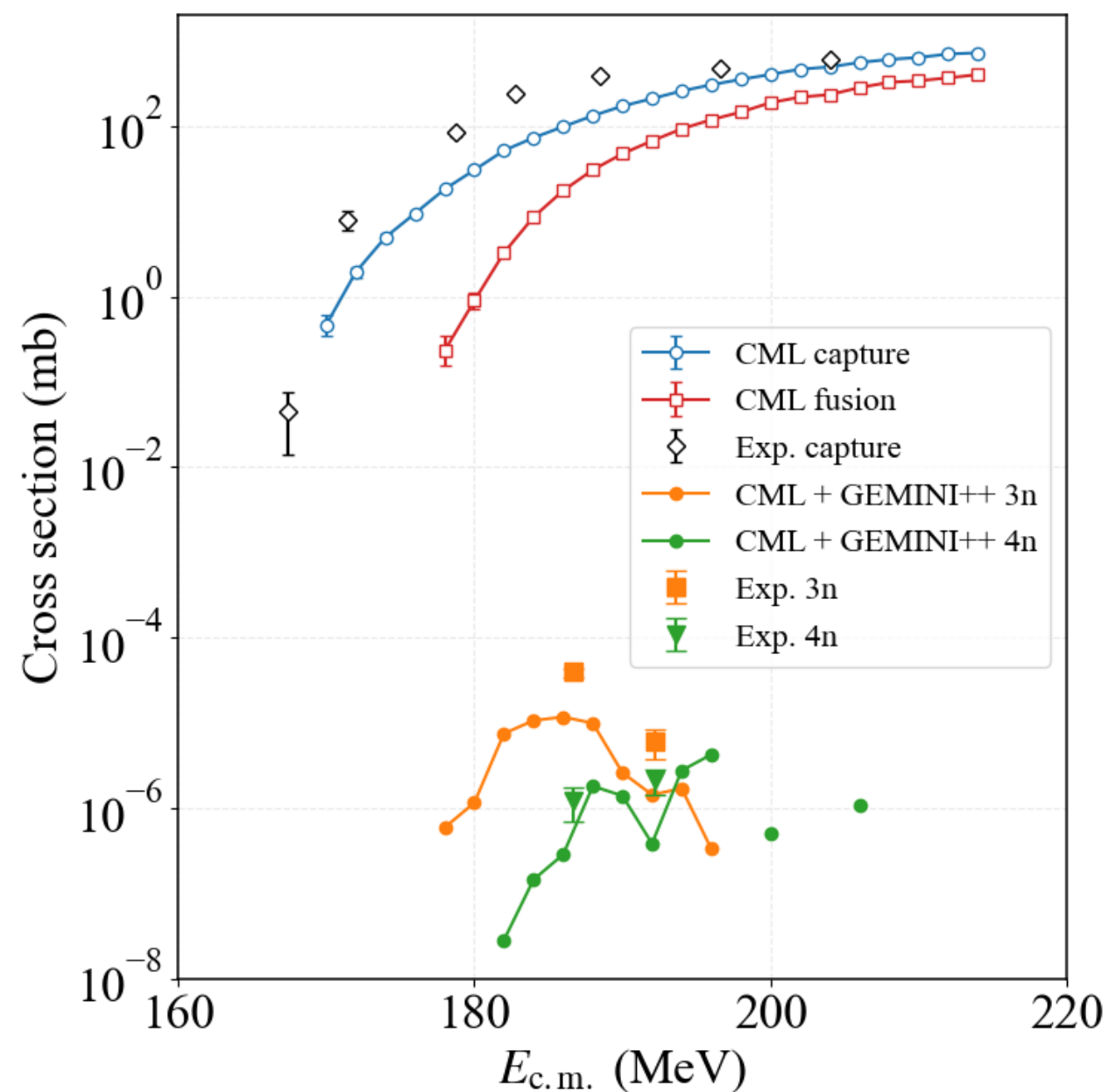


Fusion and non-Fusion:



$$\sigma_x(E) = 2\pi \int_0^{b_{\max}} b P_x(E, b) db$$

$$P_x(E, b) = \frac{N_x(E, b)}{N_{\text{tot}}(E, b)}, \quad x = \text{cap, fus}$$



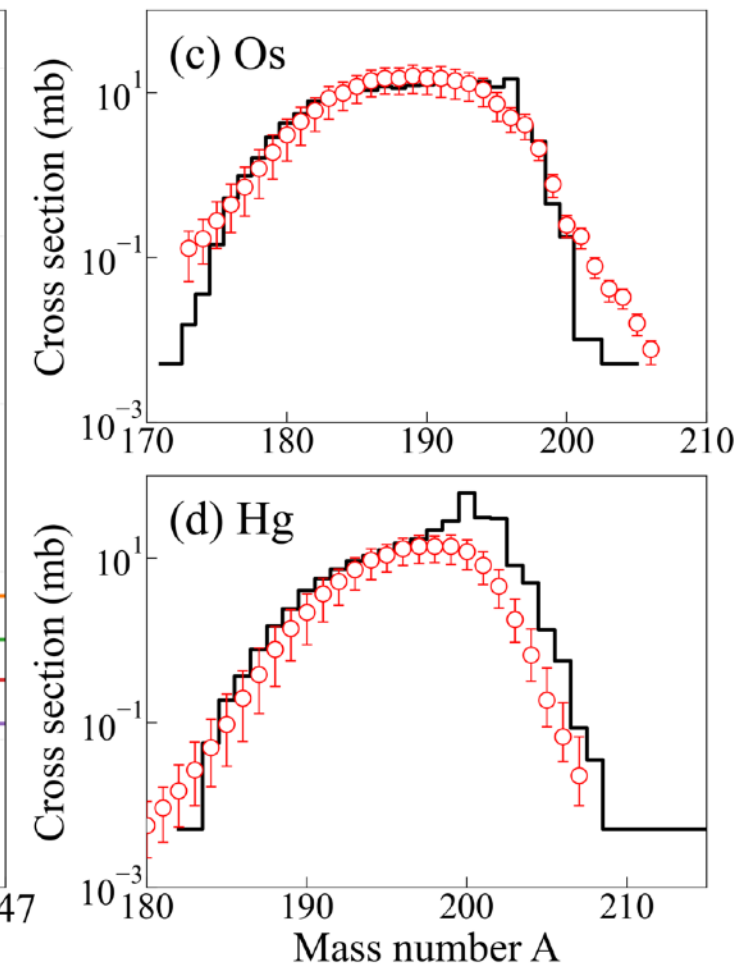
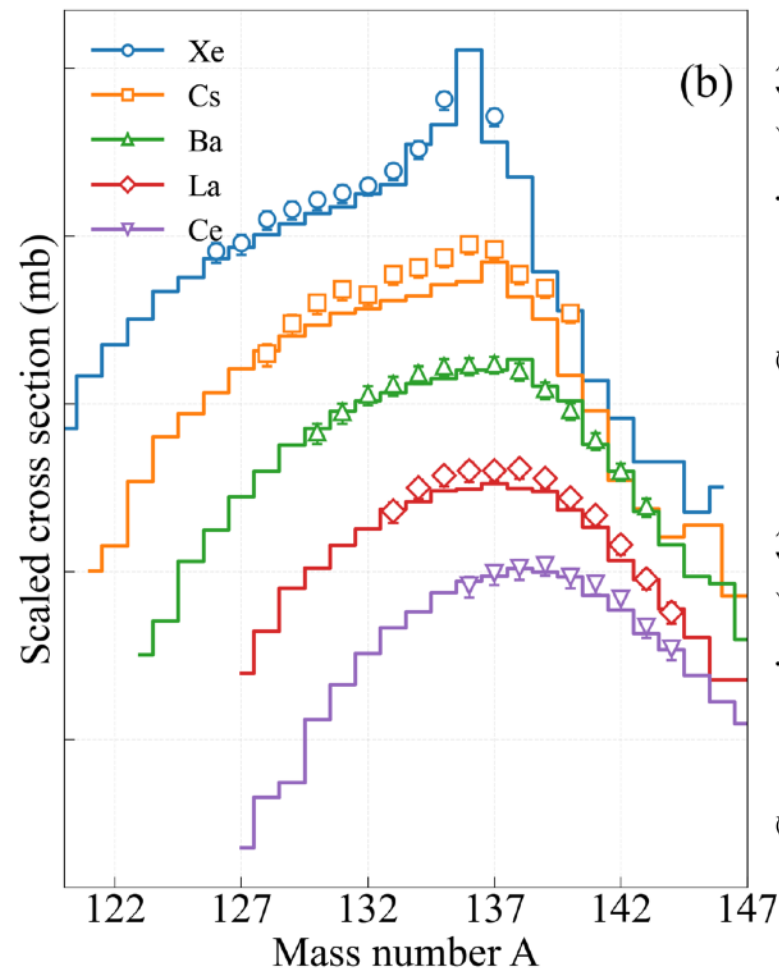
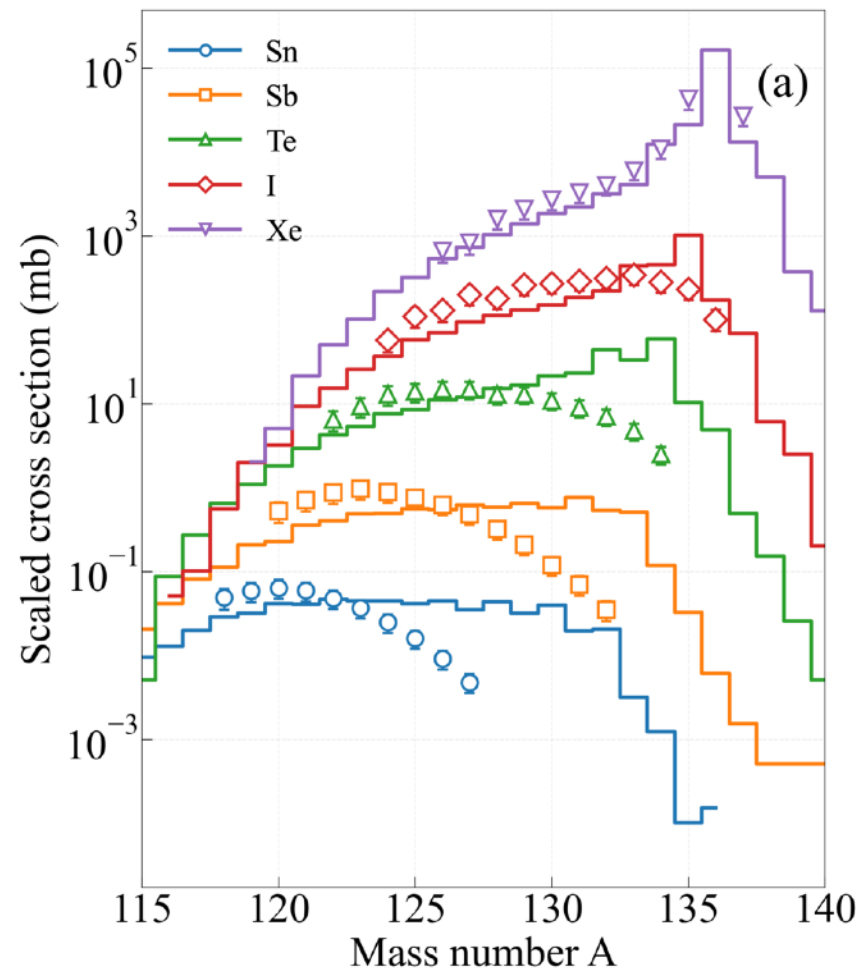
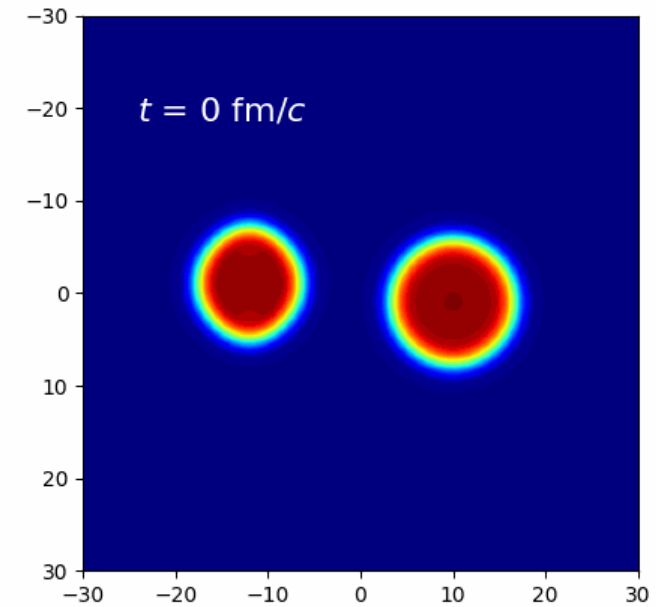
Modeling nucleus-nucleus collisions



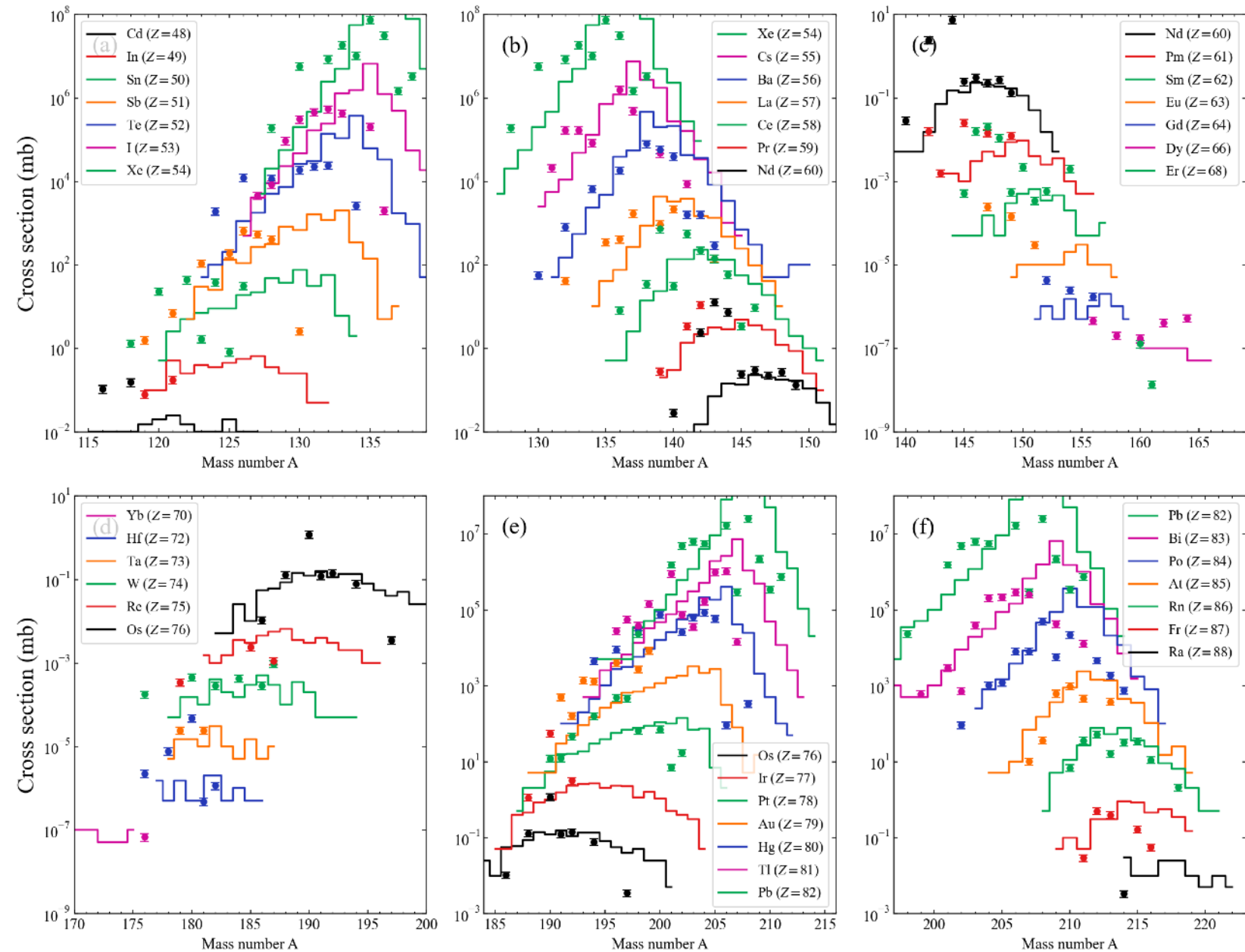
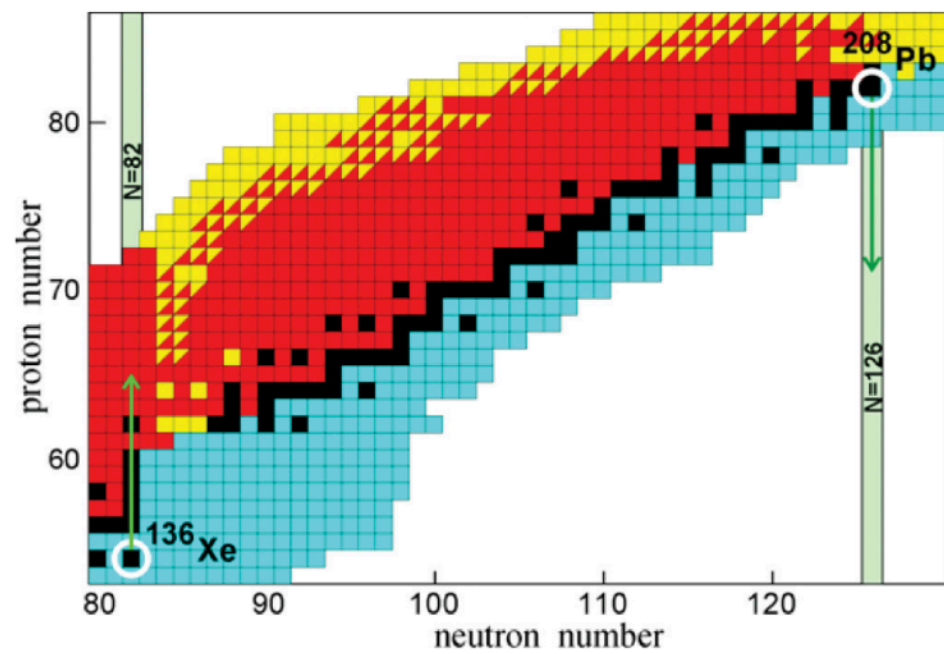
In conclusion, the promised potential of the production of new isotopes around and beyond the neutron shell $N = 126$ by multinucleon transfer reactions was established for the first time in the $^{136}\text{Xe} + ^{198}\text{Pt}$ system at an energy above the Coulomb barrier. The absolute cross

Y. X. Watanabe et al., PRL 115, 172503 (2015)

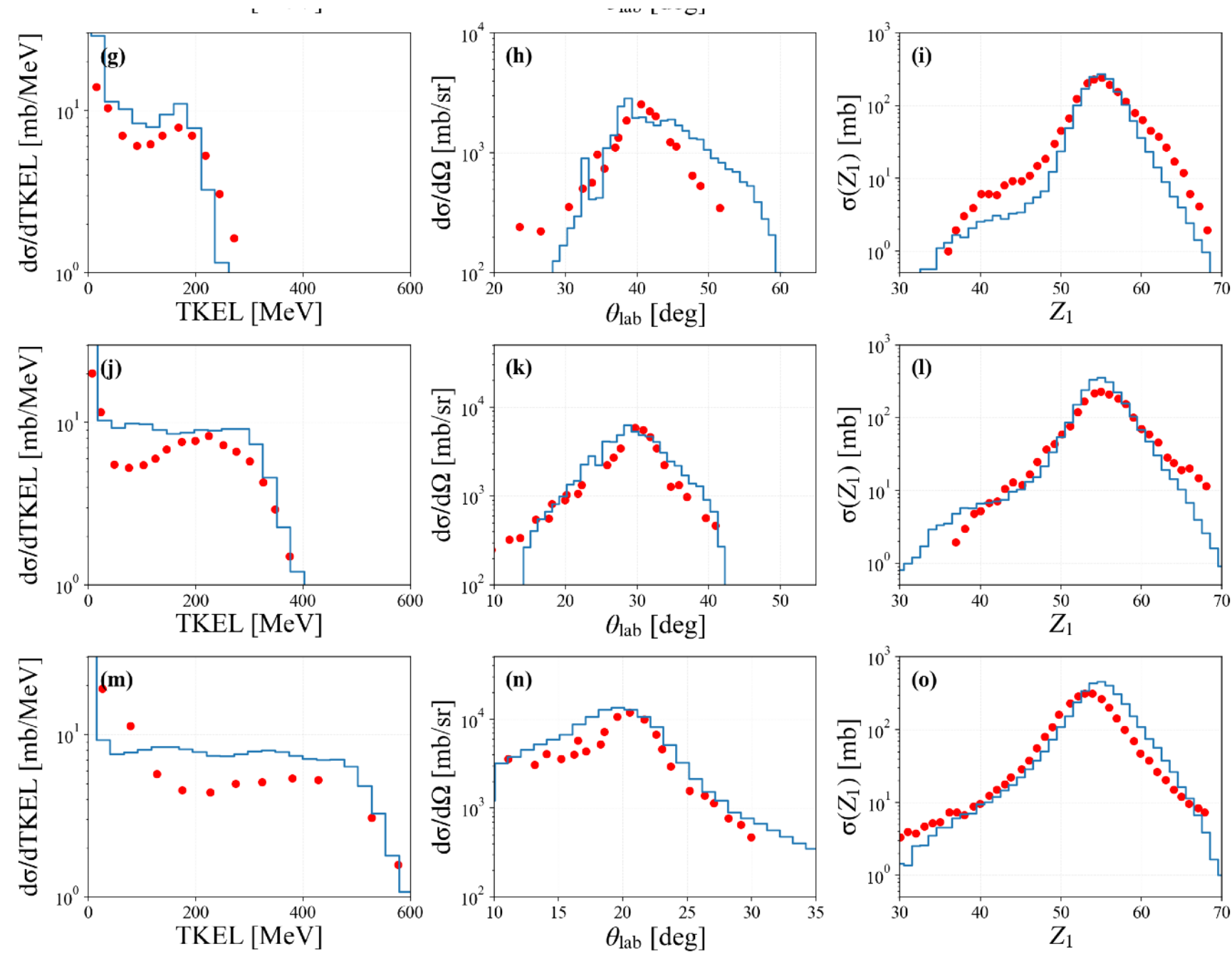
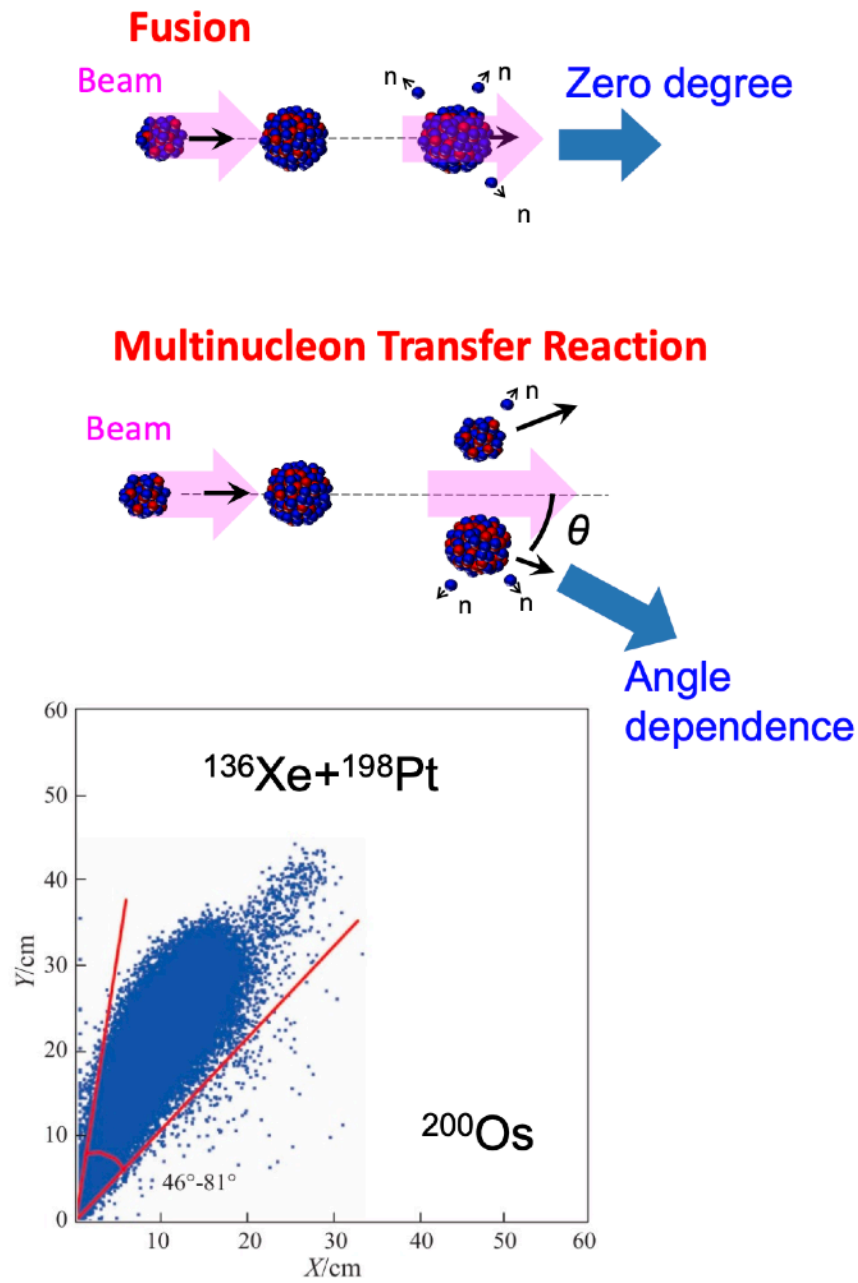
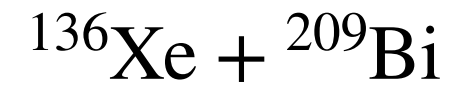
Y. X. Watanabe et al.,
Nucl. Instrum. Methods Phys. Res., Sect. B 317, 752 (2013).



Modeling nucleus-nucleus collisions $^{136}\text{Xe} + ^{208}\text{Pb}$



Modeling nucleus-nucleus collisions



Wenxue Huang et al. Nucl. Phys. Rev. (2017)

MNT · How to expand the nuclide chart

PRL **115**, 172503 (2015)

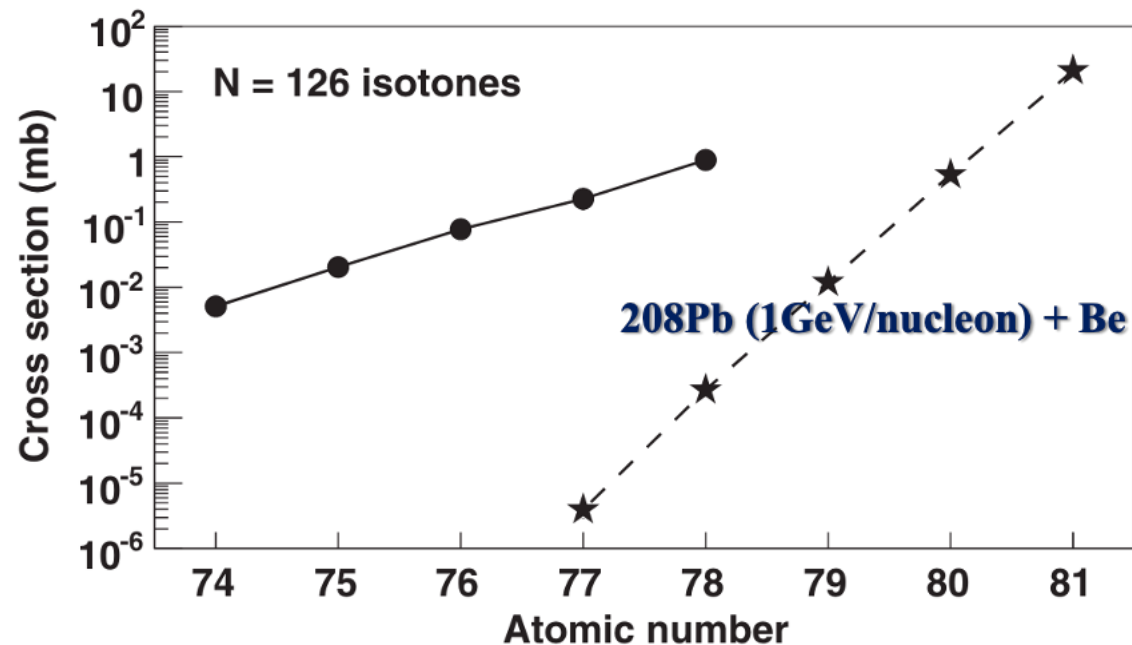
PHYSICAL REVIEW LETTERS

week ending
23 OCTOBER 2015

Pathway for the Production of Neutron-Rich Isotopes around the $N = 126$ Shell Closure

Y. X. Watanabe,^{1,*} Y. H. Kim,^{2,3,†} S. C. Jeong,^{1,‡} Y. Hirayama,¹ N. Imai,^{1,§} H. Ishiyama,^{1,‡} H. S. Jung,¹ H. Miyatake,¹ S. Choi,^{2,3} J. S. Song,^{2,3,4} E. Clement,⁵ G. de France,⁵ A. Navin,^{5,||} M. Rejmund,⁵ C. Schmitt,⁵ G. Pollaro,⁶ L. Corradi,⁷ E. Fioretto,⁷ D. Montanari,⁸ M. Niikura,^{9,¶} D. Suzuki,^{9,**} H. Nishibata,¹⁰ and J. Takatsu¹⁰

¹*Institute of Particle and Nuclear Studies, High Energy Accelerator Research Organization (KEK), Tsukuba, Ibaraki 305-0801, Japan*

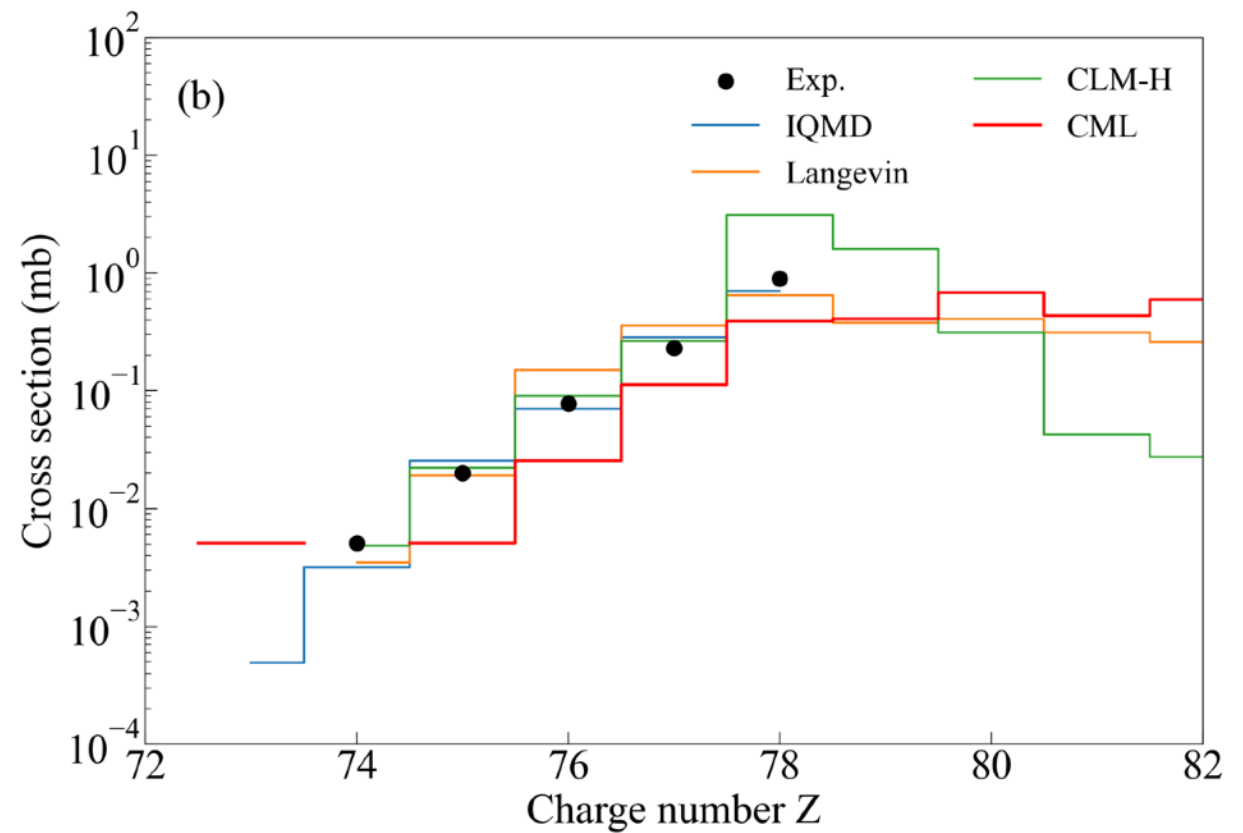
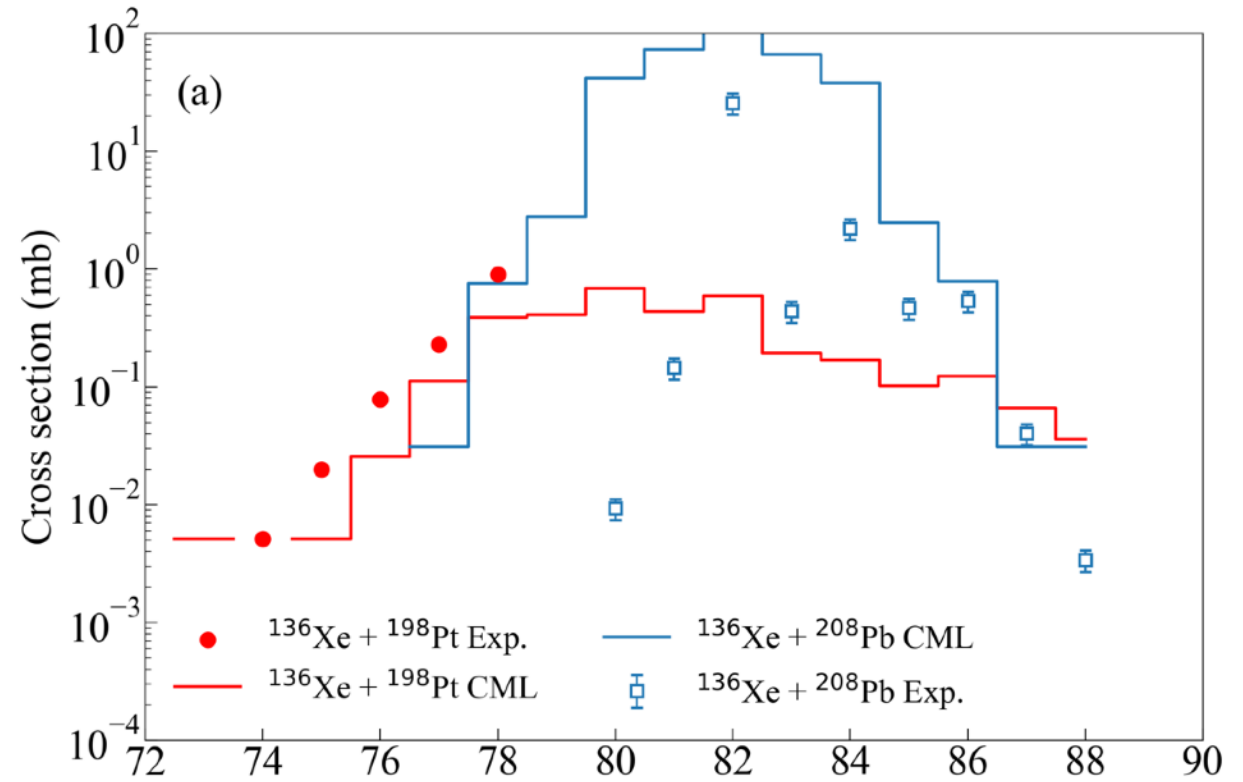


In conclusion, the promised potential of the production of new isotopes around and beyond the neutron shell $N = 126$ by multinucleon transfer reactions was established for the first time in the $^{136}\text{Xe} + ^{198}\text{Pt}$ system at an energy above the Coulomb barrier. The absolute cross

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Nucl. Instrum. Methods Phys. Res., Sect. B **317**, 752 (2013).

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Summary

- In this work, the coupled master–Langevin (CML) model has been developed to describe fusion and multinucleon transfer within a common dynamical framework.
- The calculations further support $^{136}\text{Xe}+^{198}\text{Pt}$ as a favorable reaction for producing neutron-rich $N = 126$ isotones.

Thank you for listening.

ご清聴ありがとうございました。

Potential energy surface - Adiabatic potential energy

The standard macro-microscopic model is often used for calculation of the total mass:

$$M(A, Z; q) = M_{\text{mac}}(A, Z; q) + \delta E(A, Z; q) .$$

- $M_{\text{mac}}(A, Z; q)$ is macroscopic liquid-drop mass
- $\delta E(A, Z; q)$ is the microscopic shell correction by Strutinsky shell-correction method

Nucleus	E_{mac}	E_{mic}	Mass	$E_{\text{mac}}^{\text{Moller}}$	$E_{\text{mic}}^{\text{Moller}}$	$\text{Mass}^{\text{Moller}}$	Mass^{exp}
^{40}Ca	-37.59	-5.12	42.71	-37.59	2.25	-35.34	-34.85
^{48}Ca	-43.54	-1.59	45.13	-43.54	0.11	-43.43	-44.21
^{132}Sn	-65.14	-12.32	77.46	-65.15	-11.55	-76.70	-76.61
^{136}Xe	-80.43	-6.86	87.29	-80.44	-7.20	-87.64	-86.43
^{208}Pb	-8.72	-11.82	20.54	-8.74	-12.84	-21.58	-21.77

the standard macro-microscopic approach and, in particular, to the case of highly deformed mononucleus and/or two separated nuclei leads to an incorrect result

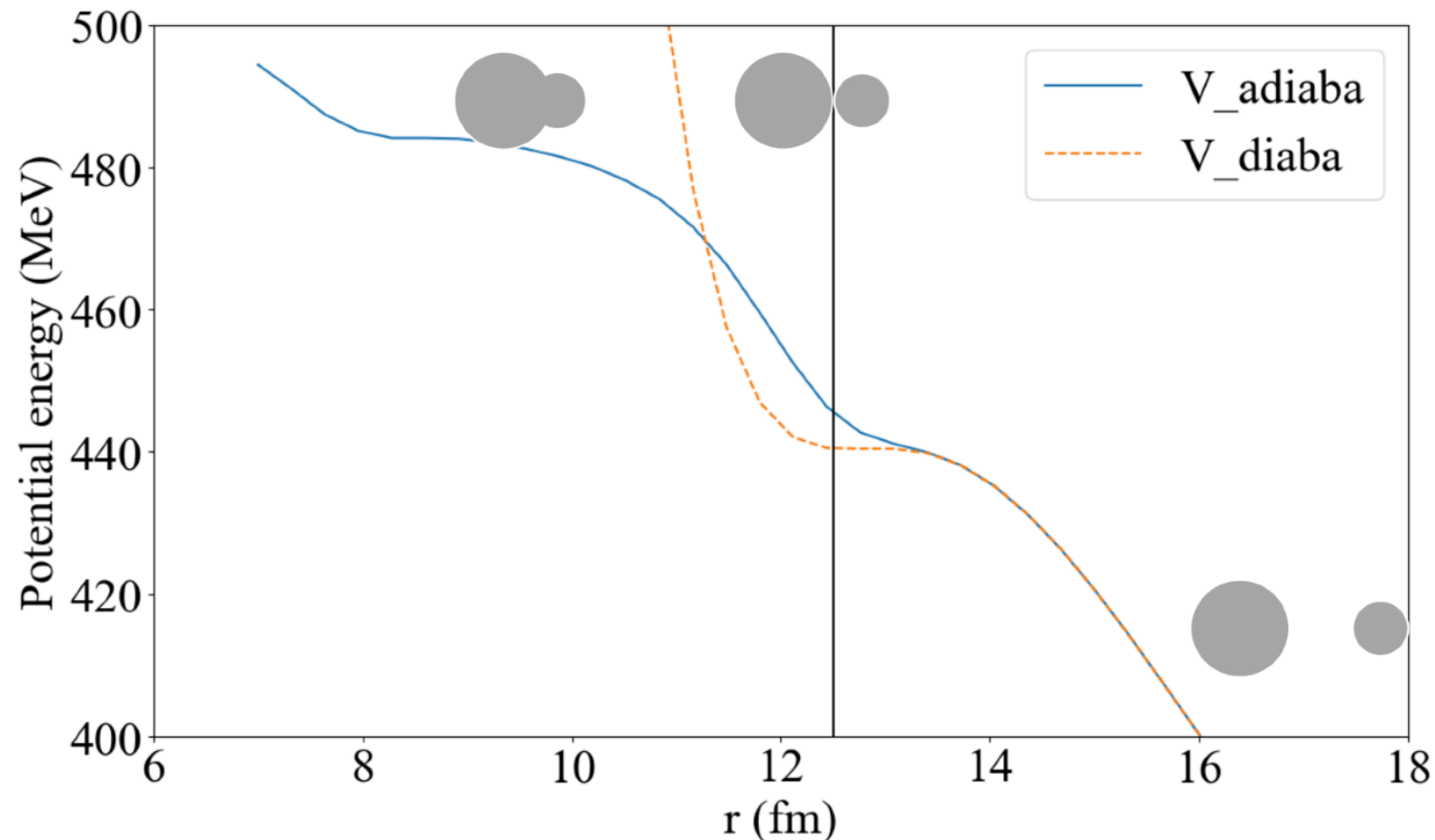
Potential energy surface - Adiabatic potential energy

In particular, the adiabatic potential energy can be calculated as follows:

$$V_{\text{adiab}}(A, Z; q) = V_{\text{standard}}^{\text{standard}}(A, Z; q)B(q) + V_{\text{diab}}(A, Z; q)[1 - B(q)]$$

$$B(q) = [1 + \exp(\frac{r - R_{\text{contact}}}{a_{\text{diff}}})]^{-2}, \quad a_{\text{diff}} = 0.5\text{fm}$$

where $V_{\text{standard}}^{\text{standard}}(A, Z; q)$ is the adiabatic potential energy calculated within the standard macro-microscopical approach.



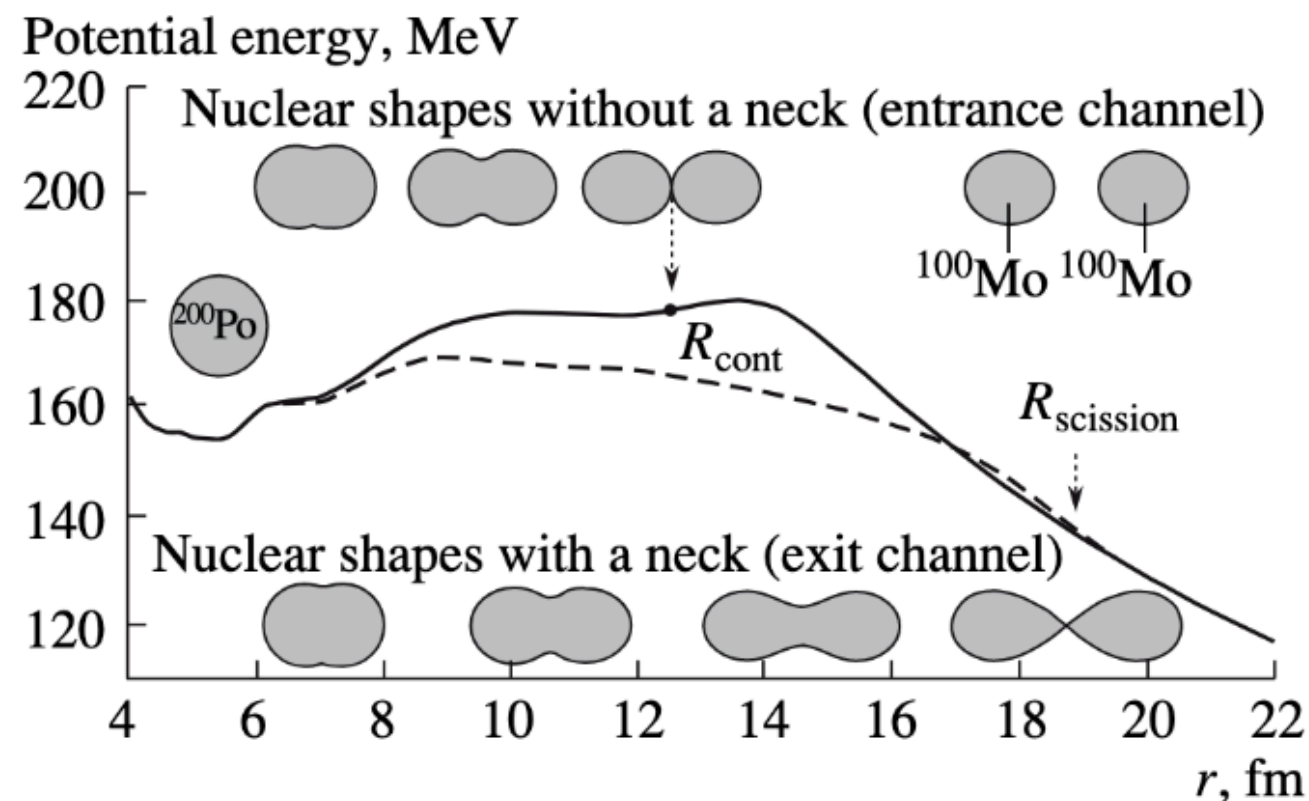
Potential energy surface - Adiabatic potential energy

The time evolution of the neck degree of freedom ε is considered in the same way as the transition from the diabatic to the adiabatic regime.

$$V(q, t) = V_{\text{adiab}}(q, \varepsilon = 1.0)f(t) + V_{\text{adiab}}(q, \varepsilon = 0.35)[1 - f(t)]$$

$$f(t) = \exp\left(-\frac{t}{\tau_\varepsilon}\right), \quad \tau_\varepsilon = 10^{-22}\text{s}$$

The entrance and the exit channel potentials are calculated with $\varepsilon = 1$ (no neck) and with $\varepsilon = 0.35$, respectively.



Equations of motion and nucleon transfer

$$W^{\pm} = \lambda_0 \sqrt{\frac{\rho(A \pm 1)}{\rho(A)}} P_{tr}(R, \beta, A \rightarrow A \pm 1)$$


Diffusion constant

It can be qualitatively assumed to be related to the size of the system and the temperature.

Therefore, we treat it as a free parameter.

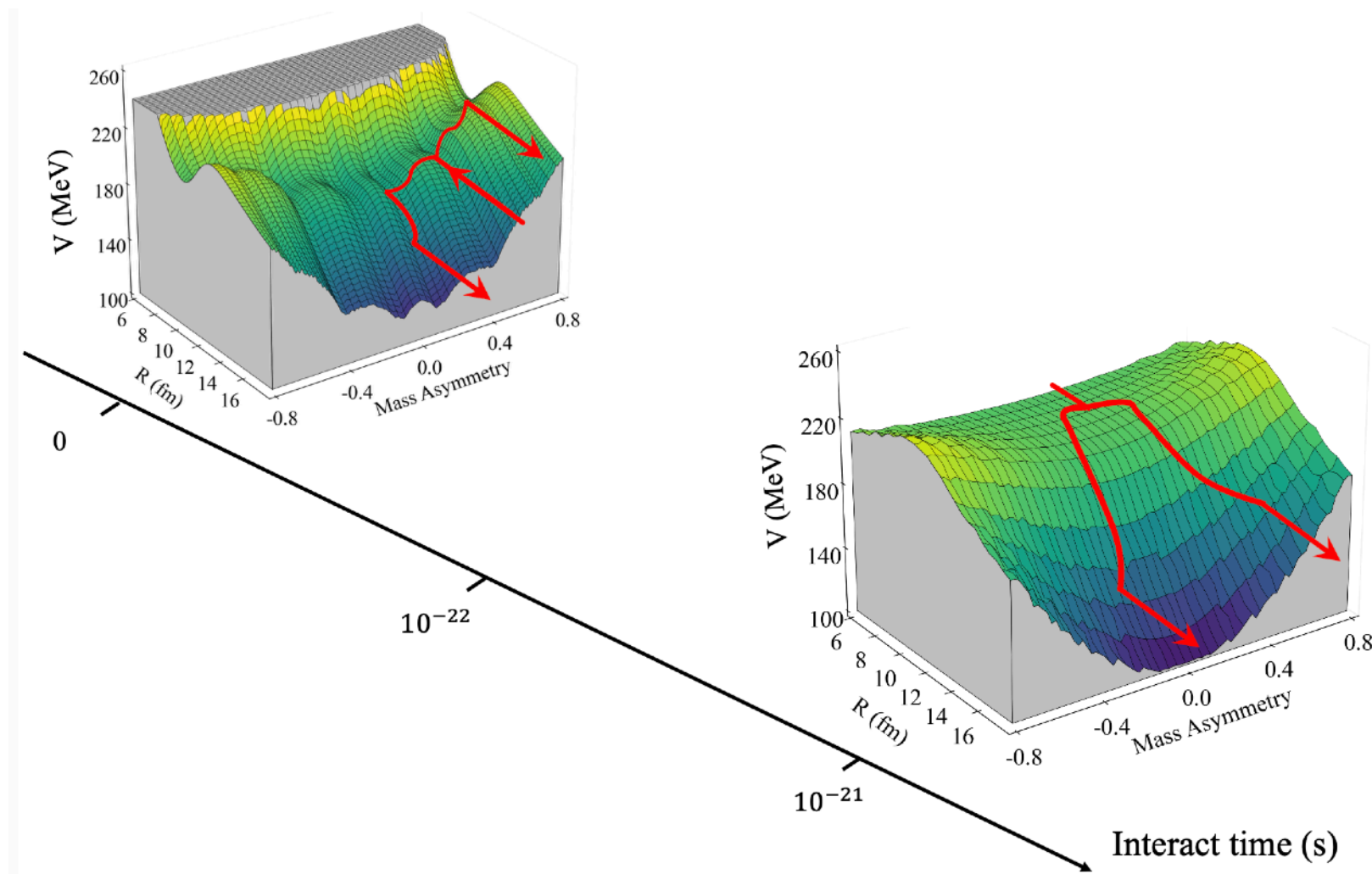
	Value	Ref
Moretto et. al.	$2\pi k \frac{R_1 R_2}{R_1 + R_2}, \quad k = 10^{21}(\text{unit/fm/s})$	Phys. Lett. B (1975) 58 26
Zagrebaev et. al.	$0.1 * 10^{22}(\text{unit/s})$	Nucl. Phys. A (2015) 944, 257.
Karpov et. al.	$5 * A_{\text{tot}} * (T/\text{MeV}) * 10^{16}(\text{unit/s})$	Eur. Phys. J. A (2022) 58:41



This value is adopted in the model

V. Zagrebaev, A. Karpov, Y. Aritomo, M. Naumenko, and W. Greiner, Phys. Part. Nucl. 38, 469 (2007).

Potential energy surface



During the nuclear reaction process, we also considered the temperature effect

$$E_{\text{mic}}(q, T) = E_{\text{mic}}(q, T = 0) \times \exp\left(\frac{-aT^2}{E_d}\right), \quad E_d = 20\text{MeV}$$

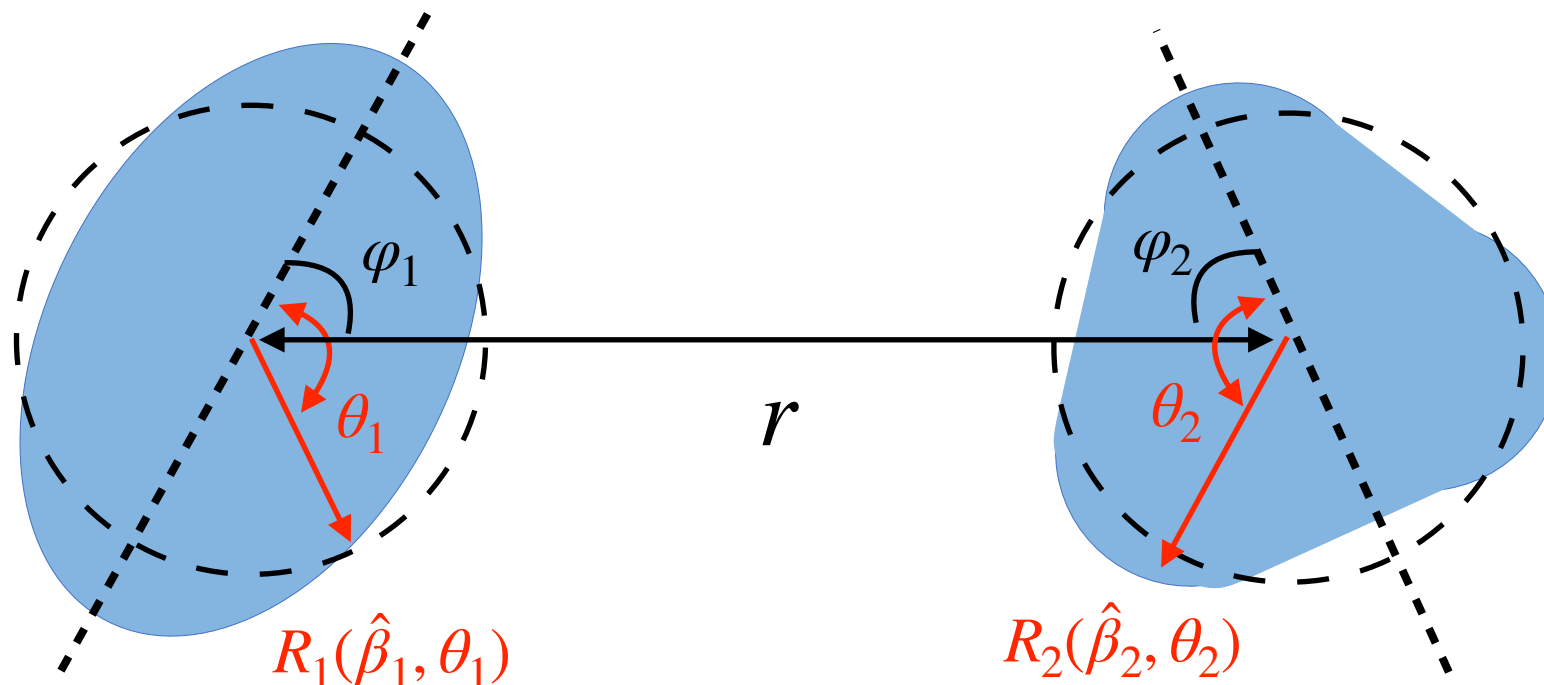
Potential energy surface - Diabatic potential energy

Redistribution of nucleons between separated nuclei changes their binding energies. The diabatic potential energy is defined as follows:

$$V_{\text{diab}}(A, Z; q) = \underbrace{V_{12}(A, Z; q)}_{\text{The interaction energy of the nuclei}} + \underbrace{M(A_1, Z_1; \delta_1)}_{\text{the masses of the reaction fragments}} + \underbrace{M(A_2, Z_2; \delta_2)}_{\text{the masses of the reaction fragments}} - \underbrace{M(A_T, Z_T; \delta_T^{\text{g.s.}})}_{\text{the masses of the entrance channel}} - \underbrace{M(A_P, Z_P; \delta_P^{\text{g.s.}})}_{\text{the masses of the entrance channel}}.$$

The interaction energy of the nuclei

the masses of the entrance channel



Potential energy surface - Diabatic potential energy

We apply a phenomenon procedure to its calculation:

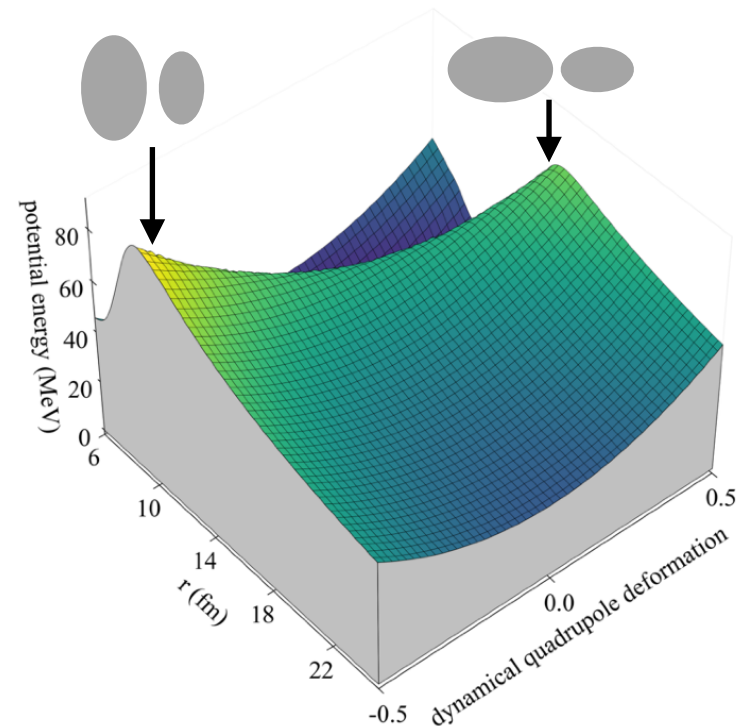
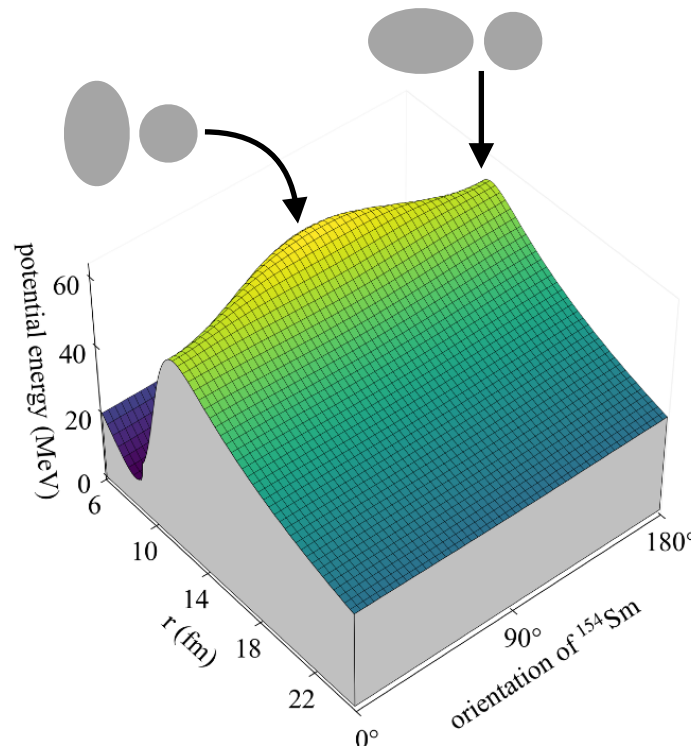
$$V_{12}(A, Z; q) = V_{12}^C(A, Z; q) + V_{12}^N(A, Z; q) + V^{\text{cent}}(A, Z; r)$$

Coulomb potential:

$$V_{12}^C(A, Z; q) \approx V_C(A, Z; r, \beta_P^0, \beta_T^0, \theta_P, \theta_T) = \frac{Z_1 Z_2 e^2}{R} + \left(\frac{9}{20\pi} \right)^{\frac{1}{2}} \left(\frac{Z_1 Z_2 e^2}{R^3} \right) \\ \times \sum_{i=P,T} R_{0i}^2 \beta_i^0 P_2(\cos \theta_i) + \left(\frac{3}{7\pi} \right) \left(\frac{Z_1 Z_2 e^2}{R^3} \right) \times \sum_{i=P,T} R_{0i}^2 [\beta_i^0 P_2(\cos \theta_i)]^2,$$

Nuclear potential:

$$V_{12}^N(A, Z; q) \approx V_N(A, Z; r, \beta_P^0, \beta_T^0, \theta_P, \theta_T) = \frac{-V_0}{1 + \exp [(r - R_P - R_T)/a]}$$



In the future, I would like to use double folding procedure